A RESTRICTED CLASS OF CONVEX FUNCTIONS

CECIL L. WOODS

1. **Introduction.** The relationship of the inequalities

$$I(f,\ x,\ h,\ \alpha) \leqq A(f,\ x,\ h,\ \beta) \tag{1.1}$$

and

$$I(f,\ x,\ h,\ \alpha) \geqq A(f,\ x,\ h,\ \beta) \tag{1.2}$$

to general convex and concave functions respectively has been treated by Radó [1].[1] In his and in this paper, $f(x)$ denotes a positive continuous function defined on a given open interval $x_1 < x < x_2$. In $I(f,\ x,\ h,\ \alpha)$ and $A(f,\ x,\ h,\ \beta)$, defined by Radó [1, pp. 267, 268] as follows:

$$(1.3)\quad \begin{aligned} I(f,\ x,\ h,\ \alpha) &= \left[\frac{1}{2h}\int_{-h}^{h} f(x+\xi)^{\alpha} d\xi\right]^{1/\alpha}, \qquad \text{if } \alpha \neq 0, \\ I(f,\ x,\ h,\ 0) &= \exp\left[\frac{1}{2h}\int_{-h}^{h} \log f(x+\xi) d\xi\right], \end{aligned}$$

$$(1.4)\quad \begin{aligned} A(f,\ x,\ h,\ \beta) &= \left[\frac{f(x-h)^{\beta} + f(x+h)^{\beta}}{2}\right]^{1/\beta}, \qquad \text{if } \beta \neq 0, \\ A(f,\ x,\ h,\ 0) &= \exp\left[\frac{\log f(x-h) + \log f(x+h)}{2}\right] \\ &= [f(x-h)f(x+h)]^{1/2}, \end{aligned}$$

x and h satisfy the inequalities $x_1 < x-h < x+h < x_2$; α and β are real exponents.

To enable us to express his definitions and results concisely, we define four classes of $f(x)$ as follows:

Let K be the class of all $f(x)$ which are convex on $x_1 < x < x_2$.

Let $K_{\alpha\beta}$ be the class of all $f(x)$ which satisfy the inequality (1.1).

Let K^* be the class of all $f(x)$ which are concave on $x_1 < x < x_2$.

Let $K^*_{\alpha\beta}$ be the class of all $f(x)$ which satisfy the inequality (1.2).

Then the four sets of pairs (α, β) which Radó defined [1, pp. 269, 281] can be defined by the statements:

Let E be the set of all pairs (α, β) for which $K \subset K_{\alpha\beta}$.

Let $\bar{E}$ be the set of all pairs (α, β) for which $K_{\alpha\beta} \subset K$.

Let E^* be the set of all pairs (α, β) for which $K^* \subset K^*_{\alpha\beta}$.

Received by the editors August 24, 1945.

[1] Numbers in brackets indicate the references at the end of the paper.