128. D. H. Lehmer: *On Ramanujan's numerical function $\tau(n)$.*

Ramanujan's numerical function $\tau(n)$ may be defined by $\sum \tau(n)x^{n-1} = \{\pi(1-x)^k\}^{24}$. Among several unsolved questions about $\tau(n)$ is the so-called Ramanujan hypothesis to the effect that if p is a prime $|\tau(p)\cdot p^{-11/2}| < 2$, which Ramanujan verified for primes $p < 30$. The present writer, in attempting to disprove this important hypothesis, has examined all primes $p < 300$, as well as $p = 571$, and finds that in all these 47 cases the hypothesis holds true. It nearly fails for $p = 103$ when $\tau(103)\cdot 103^{-11/2} = -1.918\cdots$. In connection with this hypothesis Rankin proved in 1939 that as $x \to \infty$, $x^{-12}\sum_{n \leqq x}\tau(n)^2$ tends to a limit represented by a certain double integral extended over the fundamental region of the full modular group. In this paper a practical method is devised for evaluating this integral whose value is found to be $.0320047918814\cdots$. Various congruence and divisibility properties of $\tau(n)$ are also discussed. For example $\tau(n)$ is composite for $1 < n < 7921$. (Received March 26, 1943.)

129. A. E. Ross: *Positive quaternary quadratic forms representing all integers with at most k exceptions.*

In this paper it is shown that there are a finite number of classes of positive quaternary quadratic forms which represent all integers with at most k exceptions. The determinants of such forms have an upper bound B_k depending on k. This is a generalization of the results of Ramanujan (Proc. Cambridge Philos. Soc. vol. 19 (1917) pp. 11–21), Ross (Proc. Nat. Acad. Sci. U.S.A. vol. 18 (1932) p. 607) and Halmos (Bull. Amer. Math. Soc. vol. 44 (1938) pp. 141–144). Ross gives $B_0 = 112$ for the classic case and Halmos' results imply that $B_1 \geqq 240$ in the classic case. (Received March 26, 1943.)

130. L. I. Wade (National Research Fellow): *Transcendence properties of the Carlitz ψ-function.*

The paper is concerned with quantities transcendental over the field $GF(p^n, x)$. For the Carlitz ψ-function (L. Carlitz, Duke Math. J. vol. 1 (1935) pp. 137–168) and its inverse $\lambda(t)$ the following theorem is proved. If β is algebraic (over $GF(p^n, x)$) and irrational and if $\alpha \neq 0$ is algebraic, then $\psi(\beta\lambda(\alpha))$ is transcendental over $GF(p^n, x)$. In a sense this is an analogue of Hilbert's seventh problem for the transcendence of $\alpha^\beta = c^{\beta \log \alpha}$ over the rational number field. (Received March 26, 1943.)

Analysis

131. L. W. Cohen: *On linear equations in Hilbert space.*

If the rows of the infinite matrix $A = \|a_{ij}\|$ are points in Hilbert space and $a_{j_1\cdots j_m}^{i_1\cdots i_m}$ are the m-rowed determinants with elements in A, it is shown that det $A_{i_1\cdots i_m}B'_{i_1\cdots i_m} = \sum_{j_1\cdots j_m} a_{j_1\cdots j_m}^{i_1\cdots i_m} b_{j_1\cdots j_m}^{i_1\cdots i_m}$ where $A_{i_1\cdots i_m}$, $B_{i_1\cdots i_m}$ are m-rowed minors of A, B respectively. The series is summed over all combinations of integers $j_1, \cdots, j_m$ and converges absolutely. This identity is used to establish sufficient conditions in order that the linear system represented by $Ax = y$ have a solution in Hilbert space for all y in that space. (Received March 24, 1943.)

132. R. J. Duffin: *Some representations for Fourier transforms.*

Let $\phi(x)$ be an arbitrary function and let $f(x)$ and $g(x)$ be defined by the series: $f(cx) = \sum_1^\infty (-1)^n \phi((2n-1)/x)/x$, $g(cx) = \sum_1^\infty (-1)^n \phi(x/(2n-1))/(2n-1)$. Then if