COURANT AND HILBERT ON PARTIAL DIFFERENTIAL EQUATIONS

Methoden der mathematischen Physik. Vol. 2. By R. Courant and D. Hilbert. (Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen, vol. 48.) Springer, Berlin, 1937. 16+544 pp.

Thirteen years have elapsed between the publication of the first volume of Courant-Hilbert, *Methoden der mathematischen Physik*, and this concluding second volume. The two volumes are a beautiful, lasting, and impressive monument of what Courant, inspired by the example of his great teacher Hilbert and supported by numerous talented pupils, accomplished in Göttingen, both in research and advanced instruction. Courant came to Göttingen at a time of enormous political and economic difficulties for Germany, on a difficult inheritance, with the day of the heroes, Klein, Hilbert, and Minkowski drawing to a close. But by research and teaching, by personal contacts, and by creating and administering in an exemplary manner the new Mathematical Institute, he did all that was humanly possible to propagate and develop Göttingen's old mathematical tradition. How his fatherland rewarded him is a known story. The publication of the present volume seems to the reviewer a fitting occasion for expressing the recognition his work has earned him in the rest of the mathematical world.

The first volume treated a closed, relatively unramified field: the doctrine of eigen values and eigen functions. It met with enormous success, in particular among the physicists, because shortly after its publication these matters, due to Schrödinger's wave equation, gained an unexpected importance for quantum physics. This second volume covers the theory of partial differential equations in all of its aspects which are of importance for the problems of physics. Its table of contents is therefore necessarily much more variegated, preventing the book from attaining an equally perfect esthetic unity. But it makes up for this by its wealth of material, and it shares the first volume's high didactic accomplishments. Nowadays many mathematical books do not seem to be written by living men who not only know, but doubt and ask and guess, who see details in their true perspective—light surrounded by darkness—who, endowed with a limited memory, in the twilight of questioning, discovery, and resignation, weave a connected pattern, imperfect but growing, and colored by infinite gradations of significance. The books of the type I refer to are rather like slot machines which fire at you for the price you pay a medley of axioms, definitions, lemmas, and theorems, and then remain numb and dead however you shake them. Courant imparts an insight into a situation which has manifold aspects and develops methods without disintegrating them into a discontinuous string of theorems; and nevertheless, the essential results stand out in clear relief. Numerous interesting examples help to enliven and clarify the general theories. In still another respect I found this volume comforting: when one has lost himself in the flower gardens of abstract algebra or topology, as so many of us do nowadays, one becomes aware here once more, perhaps with some surprise, of how mighty and fruitbearing an orchard is classical analysis.

Here follows a brief characterization of what the several chapters of the book contain.

Chapter 1. The usual basic concepts, preliminaries, and isolated elementary methods, namely: Discussion of the manifold of solutions for typical examples, the partial differential equation of a given family of functions, irreducibility of systems to a