CONCERNING MAXIMAL SETS*

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1. *Introduction.* Let T be any monotone system of closed subsets of an arbitrary metric space M, that is, any closed subset of a set of the system T is itself a set of T. We shall call a set of the system T a *T-set* or a *set of type T*. We may also think of T as a property such that every closed subset of a set having this property likewise has this property.

Since the null set (0) is a subset of every set, then, whatever be the system T, the null set is always a T-set.

We first establish a general existence theorem for certain maximal sets relative to a system T.

2. THEOREM. *If N is any closed non-degenerate subset of a metric space M such that N is not disconnected by the omission of any T-set, then there exists a maximal subset (a continuum) $H(N)$ of M containing N and having this property.*

PROOF. In the first place, N is connected, since $N-(0)$ is connected and is identical with N.

Secondly, N is not a T-set, for if it were then every closed subset of N would likewise be a T-set, and clearly some closed subset of N disconnects N.

Now if H denotes the sum of all sets N_0 containing N and such that N_0 is not disconnected by the omission of any T-set, then clearly H is connected and hence $\overline{H}$ is a continuum.

We proceed to show that no T-set disconnects $\overline{H}$. Suppose on the contrary, that we have a separation $\overline{H}-T=H_1+H_2$, where T is some T-set in $\overline{H}$. Then $N-N\cdot T$ is connected and thus is contained wholly in one of the sets H_1 and H_2, say H_1. But $H\cdot H_2$ contains at least one point x, since H_2 is open in $\overline{H}$. There exists a continuum N_x in H containing $x+N$ and such that N_x is not disconnected by any T-set. But $N_x\cdot T$ is a T-set and we have a separation

$$N_x - N_x\cdot T = N_x\cdot H_1 + N_x\cdot H_2,$$

and both sets of the right hand member are non-vacuous since

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