ON THE NUMBER OF APPARENT DOUBLE POINTS ON A CERTAIN V_k^n IN AN S_{2k+1}

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Consider a k-dimensional variety, V_k^n, of order n which is the locus of a single infinity of $(k-1)$-spaces in an S_{2k+1}. It is known that such a V_k^n, if it is rational, that is, if its section by a general S_{k+2} of S_{2k+1} is a rational curve, has*

$$b_k = \tfrac{1}{2}(n-k)(n-k-1)$$

apparent double points.† The question arises: What is the value of b_k when V_k^n is not rational? The case $k=1$ is familiar; a curve of order n in an S_3 has

$$b_1 = \tfrac{1}{2}(n-1)(n-2) - p \tag{1}$$

apparent double points, where p is the deficiency of the curve. It is also known that, for $k=2$, the number of apparent double points on a ruled surface F^n of order n in an S_5 is‡

$$b_2 = \tfrac{1}{2}(n-2)(n-3) - 3p, \tag{2}$$

where p is the deficiency of the curve of intersection of F^n by a general S_4 of S_5. For $k>2$, the number b_k of apparent double points of a V_k^n in an S_{2k+1} seems to be as yet unknown. It is our purpose in this note to derive a formula for this number.

Now let V_k^n be intersected by a general S_{k+2} of S_{2k+1} in a curve C^n of deficiency p. If $p>0$, we say that V_k^n is not rational. We shall say that p is also the deficiency of V_k^n and shall regard n and p as the two essential characteristics of the variety as all its other characteristics can be expressed in terms of them for a

* B. C. Wong, *On the number of* $(q+1)$*-secant* S_{q-1}*'s of a certain* V_k^n *in an* $S_{qk+q+k+1}$, this Bulletin, vol. 39, pp. 392–394.

† By an apparent double point of a V_k^n we mean a secant line of V_k^n passing through a given point of S_{2k+1}. The projection in an S_{2k} of V_k^n will have b_k improper double points each of which is the projection of an apparent double point of V_k^n.

‡ Severi, *Intorno ai punti doppi impropri di una superficie generale dello spazio a quattro dimensioni, e a'suoi punti tripli apparenti*, Rendiconti di Palermo, vol. 15 (1901), pp. 33–51.