

SPACES ADMITTING COMPLETE ABSOLUTE PARALLELISM*

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At the Colloquium of this Society held at Ithaca in September, 1925, I set forth, under the title *The New Differential Geometry*, certain developments which had taken place during the preceding ten years, growing out of the concept of infinitesimal parallelism for Riemannian spaces proposed by Levi-Civita. When these lectures were published in book form [1]† in 1927, the book included also material which had been developed in the interim. Since that time there have been many further developments. Instead of trying to make a full survey of these, I have decided to limit my paper to the theory of linearly connected manifolds admitting a complete absolute parallelism.

Levi-Civita [2] introduced the concept of parallelism in a Riemannian space as a means of giving an invariantive interpretation to the curvature of the space. Since a Riemannian space of n dimensions, V_n , may be thought of as a sub-space of a euclidean space of suitable dimensions, Levi-Civita applied the concept of parallelism of the euclidean space to vectors tangential to the sub-space. In fact, vectors a and a' at two nearby points P and P' were defined to be parallel, if the angles between a and a tangent vector f at P and a' and f are equal from the euclidean point of view for every tangential vector f . Analytically this leads to the result that, if in terms of general coordinates x^i in V_n the coordinates of P and P' are x^i and $x^i + dx^i$, then ξ^i and $\xi^i + d\xi^i$ are the components of parallel directions at P and P' , provided

$$(1) \quad d\xi^i + \left\{ \begin{matrix} i \\ jk \end{matrix} \right\} \xi^j dx^k = 0, \quad (i, j, k = 1, \dots, n),$$

where $\left\{ \begin{matrix} i \\ jk \end{matrix} \right\}$ are the Christoffel symbols of the second kind

* An address delivered at Atlantic City, December 28, 1932, as the retiring presidential address, before the American Mathematical Society.

† Such references are to the items in the bibliography at the end of this article.