tains the connected set  $W - W \times C'$ . But since we also know that  $M = W' = (W - W \times C')' + C'$ ,  $(W - W \times C')' = M$  and so  $Z_1$ contains  $Z_2$ , which is impossible. Therefore W - C is connected and so W is the sum of two mutually exclusive connected subsets, which is a contradiction. Hence W must be widely connected.

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## QUADRILATERALS INSCRIBED AND CIRCUMSCRIBED TO A PLANE CUBIC\*

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In a paper by M. W. Haskell<sup>†</sup> the geometrical configurations of triangles inscribing and circumscribing a plane cubic curve have been studied by analytic methods. The purpose of this paper is to examine the properties of quadrilaterals inscribing and circumscribing a plane cubic curve by means of elliptic functions.

The coordinates of a point on the curve will be expressed in terms of Weierstrass' elliptic functions  $\varphi(u)$  and  $\varphi'(u)$ . It is known that 3n points of the cubic are the points of intersection of the cubic with a curve of order n if‡

(1) 
$$u_1 + u_2 + \cdots + u_{3n} \equiv 0 \mod (\omega_1, \omega_2).$$

The values of the parameters of the vertices of the quadrilaterals are obtained from a consideration of the congruences

$$2u_1 + u_2 \equiv 0$$
,  $2u_2 + u_3 \equiv 0$ ,  $2u_3 + u_4 \equiv 0$ ,  $2u_4 + u_1 \equiv 0$ ,

whence

$$15u_1 \equiv 0$$

or

$$u_1=\frac{k_1\omega_1+k_2\omega_2}{15},$$

<sup>\*</sup> Presented to the Society, November 29, 1930.

<sup>†</sup> Haskell, this Bulletin, vol. 25 (1918), p. 194.

<sup>‡</sup> Appell and Lacour, Théorie des Fonctions Elliptiques et Applications, Chap. 3.