THE FUNDAMENTAL REGION FOR A FUCHSIAN GROUP*

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1. *Introduction.* The present paper is an attempt to lay the groundwork of the theory of Fuchsian groups by basing the treatment on concepts of a very simple sort. The fundamental region to which we are led is not new. It is given by the Fricke-Klein method† under certain circumstances and is identical with that given by Hutchinson‡ in an important paper. However, we make use neither of non-euclidean geometry nor of quadratic forms, and we are able to derive the major results of the theory of Fuchsian groups in an unexpectedly simple manner.

2. *The Group.* Given a group of linear transformations with an invariant circle or straight line K', the interior of K' (or the half-plane on one side of K') being transformed into itself by each transformation of the group. We shall assume that there exists a point A, not on K', such that there are no points congruent to A in a sufficiently small neighborhood of A.

Let G be a linear transformation carrying K' into the unit circle K with center at the origin and carrying A to the origin. Let S be any transformation of the group; then the set of transformations

$$T = GSG^{-1}$$

is a group with K as principal circle.§ Configurations

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* Presented to the Society, September 11, 1925.

† Fricke-Klein, *Vorlesungen über die Theorie der automorphen Funktionen*, vol. I, Chap. II.

‡ J. I. Hutchinson, *A method for constructing the fundamental region of a discontinuous group of linear transformations*, TRANSACTIONS OF THIS SOCIETY, vol. 8 (1907), pp. 261–269.

§ We use this order to mean the transformation G^{-1}, followed by S, followed by G. That is, writing $z' = S(z)$, $z' = G(z)$, etc., as the combining transformations, the new transformation is $z' = T(z) = G\{S[G^{-1}(z)]\}$.