ON IMPLICIT FUNCTIONS.

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METHODS of solving a system of m equations in $n > m$ variables, by reducing the problem to that of solving a set of differential equations, have already been given;* the aim of this paper is to present a method by which the system of m equations can be reduced immediately to a system of m differential equations of the first order, from which reduction the existence theorem and a method of constructing the solutions follow directly. For simplicity the case of two equations in four variables will be treated.

§ 1. *Reduction to a System of Differential Equations.*

It is required to solve the system

$$(1) \qquad f_1(x, y, u, v) = 0, \quad f_2(x, y, u, v) = 0$$

in the neighborhood of a set of values (x_0, y_0, u_0, v_0) for which

$$f_1(x_0, y_0, u_0, v_0) = 0, \quad f_2(x_0, y_0, u_0, v_0) = 0.$$

There is no loss in generality in assuming $x_0 = y_0 = u_0 = v_0 = 0$; suppose x, y to be the independent, u, v the dependent variables. It will be assumed that all the first partial derivatives exist and are continuous in the neighborhood of the origin defined by the inequalities

$$(R) \qquad \sqrt{x^2 + y^2} \leqq a, \quad |u| \leqq b, \quad |v| \leqq b.$$

Also, assume that the Jacobian

$$\Delta = \begin{vmatrix} \dfrac{\partial f_1}{\partial u} & \dfrac{\partial f_1}{\partial v} \\ \dfrac{\partial f_2}{\partial u} & \dfrac{\partial f_2}{\partial v} \end{vmatrix}$$

does not vanish at the origin (0, 0, 0, 0); consequently the constants a, b can be so chosen that $\Delta \neq 0$ in R. Introduce

* Horn, Gewöhnliche Differentialgleichungen beliebiger Ordnung.