AN APPLICATION OF THE NOTIONS OF "GENERAL ANALYSIS" TO A PROBLEM OF THE CALCULUS OF VARIATIONS.

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THE object of the following note is to give an illustration of the unifying power of Professor E. H. Moore's methods of "General Analysis" * by showing that a certain theorem of the calculus of variations and a certain theorem of analytic geometry are special cases of one and the same theorem of general analysis.

The theorem of the calculus of variations is the so-called fundamental lemma for isoperimetric problems,† viz.,

THEOREM I. "*If*

$$(1) \qquad \mu_0(\eta) \equiv \int_{x_1}^{x_2} [M_0(x)\eta(x) + N_0(x)\eta'(x)]\,dx = 0$$

for all functions $\eta(x)$ *which are* (*a*) *of class* C' *on* $[x_1x_2]$, (*b*) *vanish at* x_1 *and* x_2, *and* (*c*) *satisfy the* m *conditions*

$$(2) \qquad \mu_i(\eta) \equiv \int_{x_1}^{x_2} [M_i(x)\eta(x) + N_i(x)\eta'(x)]\,dx = 0 \qquad (i = 1, 2, \cdots, m),$$

then there exist m *constants* $c_1, c_2, \cdots, c_m$ *such that*

$$(3) \qquad \mu_0(\eta) + c_1\mu_1(\eta) + c_2\mu_2(\eta) + \cdots + c_m\mu_m(\eta) = 0$$

for all functions $\eta(x)$ *satisfying conditions* (*a*) *and* (*b*).

The functions $M(x)$, $N(x)$ *are supposed to be continuous on* $[x_1x_2]$.

The theorem of analytic geometry is the well known

* Compare E. H. Moore, "On a form of General Analysis with applications to linear differential equations and integral equations," Atti del IV congresso internazionale dei Mathematici, vol. 2, p. 98; and "Introduction to a form of General Analysis," in The New Haven Mathematical Colloquium, Yale University Press, New Haven, 1910.

† Compare for instance Bolza, Vorlesungen über Variationsrechnung, p. 462, footnote 1, and the references given there.