

been misled in trying to extend the circular argument of  $\cos x$ ,  $\sin x$ . This, it is true, may be taken as an arc, but it has long been known that in order to extend the analogy to the hyperbolic functions it is necessary to take as argument the ratio of the *sector* to one half the square of the radius. The baneful influence of considering trigonometric functions as lines depending on a linear argument is evidently not yet extinguished.

However, the author makes no use of his erroneous statement, but depends solely on series and De Moivre's formula, so that his formulas are correct enough.

The purpose of the work, scarcely realized in its treatment, is stated thus :

“ Nous nous proposons de rechercher si le nombre imaginaire a des lignes trigonométriques : sinus, cosinus, tangentes, circulaires ou hyperboliques. . . . Nous en établissons la trigonométrie. Nous montrons de nouveaux moyens pour résoudre certains problèmes.”

JAMES BYRNIE SHAW.

*Vorlesungen über bestimmte Integrale und die Fourierschen Reihen.*

Von J. THOMAE. Leipzig, Teubner, 1908. 8vo. vi + 182 pp. 7.80 Marks.

This book undertakes to give a rather general view of the subjects mentioned in its title. It is somewhat more on the order of a descriptive course than either a systematic development or a practical handbook. Thus, in the first fifty-seven pages the student will see unfolded before him a view of the main theorems with some regard to the dangerous places near them. He will learn that there are such functions as Euler's  $E(x)$ , Dirichlet's function, Riemann's classic function  $(x)$ , written here  $r(x)$ , Riemann's convergent series with an infinity of discontinuities

$$f(x) = r(x) + \frac{r(2x)}{2^2} + \frac{r(3x)}{3^2} + \dots$$

He will find that there is a definition of integral as the limit of a sum, which indeed is suggested by the inversion of a differentiation, and that under this definition many functions become integrable, for example those of Riemann mentioned above. The first mean value theorem he finds to hold equally for such functions, and the second mean value theorem is developed. He also finds that sometimes the variable may be changed,