1909.]

structive, or, at least, discriminating. The need of improvement in line with the constructive criticism is, in the judgment of the present reviewer, freely recognized and continually better met. H. W. Tyler.

OSGOOD'S CALCULUS.

A First Course in the Differential and Integral Calculus. Bv WILLIAM F. OSGOOD, PH.D., Professor of Mathematics in Harvard University. New York, the Macmillan Company, 1907, pp. xv + 423. Revised edition, 1909, pp. xv + 462.

PROFESSOR OSGOOD in his presidential address before the AMERICAN MATHEMATICAL SOCIETY * has discussed and illustrated the principles which his experience has led him to consider should govern the teaching of the calculus. In the present text he gives us the detailed application of those principles to the difficult pedagogical problems which confront the instructor in the first course in this subject.

Successful instruction in mathematics requires careful adjustment of the conflicting claims of rigor, formalism, and interest. Professor Osgood has recognized † that rigor is a relative matter particularly in elementary instruction, and has enunciated the principle that in such instruction a discussion is to be regarded as rigorous if it meets all the logical demands which the student can be regarded as capable of appreciating at that time. This principle is at bottom the same as that which governs contemporary judgment of productive work, and its application to instruction is but a recognition of the fact that the mathematical development of the individual differs in general from that of the race at most by a transformation of similarity.[‡] It is evident, however, that such a principle must be applied with care, for otherwise it may be cited in defense of a multitude of mathematical sins. If used with judgment, however, as is the case in this text, it becomes the very foundation of successful mathematical teaching.

The applications of this principle are in evidence throughout the book. For example: the theorem on the limit of the sum is at first (page 12) tacitly assumed, then (page 14) mentioned in a footnote, and finally proved (page 15) when the progress

^{*} BULLETIN, vol. 13 (1907), pp. 449-467. † Annals of Mathematics, ser. 2, vol. 4 (1903), p. 178. ‡ Cf. Cantor, Geschichte der Mathematik, 3 Aufl., Bd. 1, p. 3.