SINGULAR POINTS OF A SIMPLE KIND OF DIFFERENTIAL EQUATION OF THE SECOND ORDER.

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IN a series of four memoirs in the *Journal de Mathématiques* (series 3, numbers 7, 8; series 4, numbers 1, 2) Poincaré has, among other things, discussed the topology of curves defined by ordinary differential equations of a simple character. In a recent course of lectures Hilbert laid considerable stress on the importance of these results and exhibited an elegant method for obtaining them in the case of a differential equation of the form $dy/dx = (cx + dy)/ax + by)$. In the following paper I have shown how the same method can be used for an ordinary differential equation of the second order. My results tally with those of Poincaré insofar as the latter are enumerated; but they are more detailed than his and are, I think, more simply obtained.

Given (1) $d^2y/dx^2 = (dx + ey + f \cdot dy/dx)/(ax + by + c \cdot dy/dx)$, a, b, c, d, e, f real constants. Put (2) $dy/dx = z$ and (1) becomes

$$(3) \qquad dz/dx = (dx + ey + fz)/(ax + by + cz).$$

Now write

$$(4) \quad dx/dt = ax + by + cz, \quad dy/dt = z, \quad dz/dt = dx + ey + fz.$$

Multiply the second and third of these equations by m and n respectively and add,

$$(5) \quad d(x + my + nz)/dt = (a + nd)x + (b + ne)y + (c + m + nf)z.$$

Equate the second member of (5) to $\lambda(x + my + nz)$ and determine m, n and λ accordingly. We find

$$(6, 7) \qquad n = (\lambda - a)/\lambda, \quad m = (bd - ac + e\lambda)/d\lambda,$$

$$(8) \quad \lambda^3 - (a + f)\lambda^2 + (af - cd - e)\lambda - (bd - ae) = 0.$$