ON THE NUMBER OF ROOTS OF THE HYPERGEOMETRIC SERIES BETWEEN ZERO AND ONE.

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IN 1890 Klein* published his solution of the problem of enumerating the roots of the Hypergeometric Series between 0 and 1.

His method depending on the conformal property of the Schwarzian s-function, finally turns on a discussion of the shape of the circular triangles on which the x-halfplane is mapped.

Solutions by Hurwitz† and Gegenbauer‡ appeared soon after, both depending on the determination of a chain of contiguous hypergeometric functions which could be employed as a set of Sturmian functions.

Klein's method, while it makes use only of the differential equation and yields the desired result in an exceedingly neat form, does not lead to this result so directly or naturally as certain methods of Sturm (Tom. 1, *Liouville's Journal*).

It is the object of this paper to apply two well known theorems of the above mentioned memoir of Sturm to the solution of the problem in hand.

The theorems referred to are:

A. Let x_1 and x_2 be two regular singular points of the differential equation

$$\frac{d^2\overline{y}}{dx^2} = \varphi(x, a)\,\overline{y} \tag{1}$$

If there be no singular point between x_1 and x_2 and all the magnitudes involved be supposed real, the real roots of $\overline{y}_1$ between x_1 and x_2, $\overline{y}_1$ being the solution corresponding to the larger exponent of x_1, will move toward the point x_1, if, for all values of x between x_1 and x_2, $\varphi\,(x, a)$ decrease with the decrease (increase) of a; *i. e.*, $\overline{y}_1$ is gaining or *at most* not losing roots between x_1 and x_2.

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* *Math. Ann.*, vol. 37.
† *Math. Ann.*, vol. 38.
‡ *Wiener Sitzungsberichte*, vol. 100[2a].