the beauty and unity of mathematics, we must broaden our perspectives and hone our intellectual curiosity. Perhaps algebraic topologists should learn some mathematical physics, and mathematical physicists some algebraic topology; not because it is useful, but because it is interesting! It seems that Lefschetz agreed.

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Geometry of Banach spaces–Selected topics, by Joseph Diestel, Lecture Notes in Mathematics, no. 485, Springer-Verlag, Berlin, Heidelberg, New York, 1975, xi + 282 pp., $11.50.

Beginning with Banach's *Operations lineaires* the study of Banach spaces has been a pursuit of classification. This can be in terms of the classically important spaces as $C(K)$ or $L_p(\mu)$, or it can be in terms of certain desirable internal conditions on norm and specified elements such as smoothness and convexity properties, or it can be in terms of external conditions, for example, on dual spaces, subspaces, or factorization of operators. An elegant example of the first is the Bohnenblust-Kakutani result that a Banach lattice is linearly isometric to $L_p(\mu)$ or a sublattice of $C(K)$ if and only if $\|x + y\|^p = \|x\|^p + \|y\|^p$ or $\|x + y\| = \max(\|x\|, \|y\|)$ whenever $x \wedge y = 0$. Smoothness refers to the existence of unique supporting hyperplanes to points on the surface of the unit ball and is usually phrased in terms of the differentiability of the norm. Uniform convexity describes the shape of the surface of the unit ball. These concepts are important to minimization problems in optimization theory and P.D.E. among others. These conditions are "geometric" in nature and are not, in general, preserved under isomorphisms. However, deep studies have been made into various Banach space properties which imply smoothness or convexity conditions under some equivalent norm. Perhaps the deepest and most important of these is the one obtained by P. Enflo which states that a Banach space X has an equivalent norm under which it is uniformly convex if and only if it is superreflexive (i.e., every Banach space Y which has the property that every finite dimensional subspace of Y is almost isometrically embeddable into X is itself reflexive). This is an elegant blending of the internal geometry (uniform convexity) with the external geometry (superreflexivity). There are many examples of the third type mentioned above. For example, the theory of $\mathcal{L}_p$ spaces (roughly, a Banach space is a $\mathcal{L}_p$ space if it is the union of an upwards directed family of finite dimensional spaces each uniformly equivalent to an $l_p(n)$, $1 \leqslant p \leqslant \infty$). Thus there is the elegant Lindenstrauss-Pełczyński-Rosenthal result that X is a $\mathcal{L}_p$ space or a Hilbert space if and only if it is isomorphic to a complemented subspace of an $L_p(\mu)$ space, $1 \leqslant p < \infty$. In duality there is the Lindenstrauss-Rosenthal result that X is a $\mathcal{L}_p$ space if and only if X^* is a $\mathcal{L}_{p'}$ space, $1 \leqslant p \leqslant \infty$. In factorization of operators, there is the beautiful theorem of Davis, Figil, Johnson and Pełczyński that a weakly compact operator factors through a reflexive space. One also has the deeply significant work of many authors (Lindenstrauss, Pełczyński, Nikišin, Stein, Rosenthal, Maurey, and others) on absolutely p-