BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 78, Number 2, March 1972

INVARIANT SPLITTINGS IN NONASSOCIATIVE ALGEBRAS: A HOPF APPROACH[1]

BY H. P. ALLEN

Communicated by M. Gerstenhaber, August 10, 1971

The purpose of this short note is to announce generalizations of known invariant splitting theorems due to Taft [4], [5], [6], [7] and Mostow [1], which have been obtained by Hopf methods. The approach is an outgrowth of techniques developed by M. Sweedler in order to study algebraic groups from a Hopf point of view, and was motivated by several conversations with him.

0. Let (V, Δ, ε) be a coalgebra over the field k which is equipped with the structure of a unitary associative algebra by means of coalgebra morphisms $m: V \otimes_k V \to V$ and $\mu: k \to V$. $A = (V, \Delta, \varepsilon, m, \mu)$ is then a bialgebra and is a Hopf algebra if $\mathrm{id} \in \mathrm{End}_k(V)$ is invertible in the convolution structure [2, p. 71]. We will often confuse A with V.

Recall that A^* has a natural associative algebra structure relative to

$$A^* \otimes_k A^* \hookrightarrow (A \otimes_k A)^* \xrightarrow{\Delta^*} A^*, \qquad k \cong k^* \xrightarrow{\varepsilon^*} A^*.$$

An element $\lambda \in A^*$ is called a (left) integral for A if $a^*\lambda = \langle a^*, 1_A\rangle\lambda$ for all $a^* \in A^*$. If $M \xrightarrow{\psi} M \otimes_k A$ is a right A-comodule, then M carries a (rational) left A^*-module structure via

$$A^* \otimes_k M \to A^* \otimes_k M \otimes_k A \to M \otimes_k A^* \otimes_k A \to M \otimes_k k \cong M$$

[2, pp. 33–36, 91–92] and one has the adjoint A^*-module structure on $E = \mathrm{End}_k M$ given in [3, p. 332] which is characterized by the relation

$$(a^* \rightharpoonup T)(m) = \sum_{(m)} (a^* \leftharpoonup m_{(1)}) \cdot T(m_{(0)}) \quad \text{for } a^* \in A^*, T \in E \text{ and } m \in M.$$

If A has an integral λ which satisfies $\langle \lambda, 1_A\rangle = 1$, then every rational A^*-module is completely reducible. Conversely, if ${}_{A^*}A$ is a completely reducible rational A^*-module (via the regular right A-comodule structure of A) then A has an integral satisfying the above condition.

1. Let $\mathfrak{N}$ be a nonassociative algebra over k, $\mathfrak{R}$ an ideal in $\mathfrak{N}$ with $\mathfrak{R}\mathfrak{R} = \{0\}$, $\mathfrak{S}$ a subalgebra of $\mathfrak{N}$ with $\mathfrak{N} = \mathfrak{S} \oplus \mathfrak{R}$ (as vector spaces). We have

---

AMS 1970 *subject classifications*. Primary 13D99, 17A99.
Key words and phrases. Hopf algebra, invariant radical splitting.
[1] This research was partially supported by NSF grants GP 11404 and RF 3224-A1.