A COMPLETE SYSTEM FOR THE SIMPLE GROUP G_{60}^{6}

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The problem of this paper is to obtain an irreducible set of polynomials in terms of which all the polynomials that are invariant under the simple group G_{60}^{6} can be expressed as polynomial functions.

No polynomials of degrees and extents 1 and 2 respectively are invariant under this group except the respective elementary symmetric polynomials, E_1 and E_2. Hence, this irreducible set will contain no polynomials of extents 1 and 2 other than the elementary symmetric polynomials which we shall write as S-polynomials:

$$S_1 \equiv E_1 \quad \text{and} \quad S_2 \equiv E_2.$$

The group leaves invariant the set of triples,

123, 134, 145, 156, 162, 235, 346, 452, 563, 624,

and the complementary set

124, 146, 163, 135, 152, 243, 465, 632, 354, 526.

Hence the S-polynomials,

$$S_{123} \equiv x_1x_2x_3 + x_1x_3x_4 + x_1x_4x_5 + \cdots ,$$
$$S_{124} \equiv x_1x_2x_4 + x_1x_4x_6 + x_1x_6x_3 + \cdots ,$$

or, as we shall write them,

$$S_{123} \equiv 123 + 134 + 145 + \cdots ,$$
$$S_{124} \equiv 124 + 146 + 163 + \cdots ,$$

are invariant under the group. Clearly, the polynomials $S_{1^i2^j3^k}$ and $S_{1^i2^j4^k}$ are also invariant under the group, where

$$S_{1^i2^j3^k} + S_{1^i2^j4^k} \equiv \Sigma_{1^i2^j3^k},$$

the general symmetric polynomial of extent 3 on 6 variables.

Each of the 15 quadruples that can be selected from among the numbers $1, \cdots, 6$ may be regarded as the *intersection* of two of the triples in each of the sets above. Thus, 1234 is the