of rigor, and its clever handling of a pedagogically difficult subject should recommend it to teachers of mathematics.

T. H. GRONWALL.

Vorlesungen über Variationsrechnung. Von OSKAR BOLZA. Leipzig, B. G. Teubner, 1909. ix+795+10 pp.

THIS book is a revised and considerably enlarged German edition of the same author's "Lectures on the Calculus of Variations," Chicago, University Press, 1904.*

In Chapter I, entitled "The first variation in the simplest class of problems," the author, after some introductory remarks on the scope of the calculus of variations, starts by explaining his system of notations, which is exceedingly precise and consistent, although it would seem to the reviewer that a somewhat less elaborate system would have made the book easier to read without any sacrifice of rigor. The classical results in the theory of the first variation of the integral $\int_{x_1}^{x_2} f(x, y, y')dx$ with fixed and variable end points are set forth, including Euler's differential equation and Du Bois-Reymond's lemma. The proof for the latter given on page 28 is due to Hilbert; it would perhaps have been more appropriate to give the proof of Zermelo (*Mathematische Annalen*, volume 58 (1904), page 558), which is unsurpassed in simplicity, brevity, and elegance.

Chapter II, "The second variation in the simplest class of problems," contains the Legendre and Jacobi criteria; the exposition, excellent already in the English edition, is even better in the present book and stands forth as a model of clearness and precision. Chapter III, "Sufficient conditions in the simplest class of problems," deals with the conditions for a weak minimum, the construction of a field of extremals, Weierstrass's expression for the second variation in terms of the E-function, which is here introduced by means of Hilbert's invariant integral, and various conditions for the existence of a strong minimum.

Chapter IV, "Auxiliary theorems on functions of a real variable," contains various lemmas on implicit functions and existence theorems for differential equations, preparatory to Chapter V, "Weierstrass's theory of the simplest class of problems in parametric representation," which treats anew,

* Reviewed by E. R. Hedrick in BULLETIN, vol. 12 (1906), pp. 80–90.