THE HESSIAN OF THE CUBIC SURFACE.

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§1. *A special form of the Hessian, the coördinates of which are expressible in terms of hyperelliptic functions.*

Let $[\omega_i]$ $(i=1, 2, \cdots, 6)$ be a set of six simultaneous half periods (or briefly, six half periods) forming what might be called a *Weber Sextuple, i. e.*, a sextuple having no more than three half periods in common with any Rosenhain sextuple.*

Also let θ be an even theta function of order four and characteristic zero, vanishing (of the second order) for the six half periods $[\omega_i]$. Every such function can be expressed linearly in terms of four functions of the same kind. Denoting these latter by x_1, x_2, x_3, x_4, it is evident that between them exists a homogeneous algebraic equation of the fourth degree†

$$f(x_1, x_2, x_3, x_4) = 0. \tag{1}$$

Interpreting this equation geometrically as representing a quartic surface, we observe that to the remaining ten half periods correspond ten nodes on the surface.‡ Also, since there are ten theta functions of the first order

$$\vartheta_j \qquad (j=1, 2, \cdots, 10)$$

which vanish for three of the half periods $[\omega_i]$, the ten equations

$$\vartheta_j = 0 \qquad (j=1, 2, \cdots, 10) \tag{2}$$

represent ten lines on the surface f. § It is apparent that the ten nodes and the ten lines form the vertices and the edges of a complete pentahedron, and hence the surface f is the Hessian of a cubic. It is not the Hessian of the most general form of the cubic, however, since that depends on

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* Such a sextuple corresponds, as Weber has shown, to six independent nodes of the Kummer surface, from which the ten remaining nodes can be determined.

† See Humbert, "Théorie générale des surfaces hyperelliptiques," *Liouville's Journal*, 1893, p. 460.

‡ Ibid., p. 48.

§ Ibid., p. 44.