THE DECOMPOSITION OF MODULAR SYSTEMS OF RANK n IN n VARIABLES.

(Presented to the Chicago Section of the American Mathematical Society, April 24, 1897.)

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I.

THEOREM A. *If in the realm* $\mathfrak{R}$ *of integrity-rationality* $\mathfrak{R} = [x_1, \cdots, x_n]\ (\mathfrak{R}_1', \ldots, \mathfrak{R}_\nu')$, *where the* $x_1 \cdots x_n$ *are independent variables and the realm* $\mathfrak{R}' = (\mathfrak{R}_1', \cdots, \mathfrak{R}_\nu')$ *is independent of the* $x_1 \cdots x_n$, *the modular system*

$$(1) \qquad \mathfrak{L} = [L_1[x_1, \cdots, x_n], \cdots,\ L_m[x_1, \cdots, x_n]]$$

is contained in the coefficient modular system $\mathfrak{F}$

$$(2) \qquad \mathfrak{F} = [\cdots, \underset{(k_1 \cdots k_n \mid t)}{f_{k_1, \ldots k_n}}, \cdots]$$

of the form

$$(3) \qquad F[u_1, \cdots, u_n] = \sum_{k_1 \ldots k_n \mid t} f_{k_1 \ldots k_n} u_1^{k_1} \cdots u_n^{k_n}$$

$$= \prod_{h=1,s} \Big(\sum_{i=1,n} (x_i - \xi_{hi})\, u_i^{e} \Big)^h \qquad \Big(t = \sum_{h=1,s} e_h\Big)$$

where the $f_{k_1 \ldots k_n} = f_{k_1 \ldots k_n}[x_1, \cdots, x_n]$ *belong to* $\mathfrak{R}$ *and the* ξ_{hi} *belong to* $\mathfrak{R}'$ *or to a family-realm containing* $\mathfrak{R}'$, *and where the* s *linear forms* $\sum_{i=1,n} (x_i - \xi_{hi})\, u_i$ $(h = 1, 2, \cdots, s)$ *are distinct, then in the realm* $\mathfrak{R}^* = [x_1, \cdots, x_n]\ (\mathfrak{R}_1', \cdots, \mathfrak{R}_\nu', \xi_{hi}\ {}^{h=1,2,\ldots s}_{i=1,2,\ldots n})$ *the system* $\mathfrak{L}$ *decomposes (in the sense of equivalence) into relatively prime factors* $[\mathfrak{L}, \mathfrak{D}_h^{e_h}]$,

$$(4) \qquad \mathfrak{L} \sim \prod_{h=1,s} [\mathfrak{L}, \mathfrak{D}_h^{e_h}],$$

where $\qquad \mathfrak{D}_h = [x_1 - \xi_{h1}, \cdots, x_n - \xi_{hn}]$, *so that*

$$(5) \qquad [\mathfrak{D}_h, \mathfrak{D}_{h'}] \sim [1] \quad (h \neq h';\ h, h' = 1, 2, \cdots, s).$$

Every such modular system $\mathfrak{L}$ *is of rank* n *in* n *variables.*

Every modular system $\mathfrak{L}$ *of rank* n *in* n *variables decomposes in this way in particular with respect to its* resolvent *form*

$$\mathrm{F}[u_1, \cdots, u_n].$$