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FLOER HOMOLOGY AND ARNOLD CONJECTURE

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1. Introduction

Let V be a closed symplectic manifold with a symplectic form ω. This means that ω is a closed non-degenerate two-form. Because of the non-degeneracy of ω, with any time-dependent periodical Hamiltonian function $H : V \times S^1 \to \mathbf{R}$, we can associate a θ-dependent vector field X_{H_θ} given by:

$$\omega(X_{H_\theta}, \cdot) = dH_\theta,$$

where $\theta \in \mathbf{R}$ is the usual angular coordinate of S^1 and $H_\theta = H|_{S^1\times\{\theta\}}$. Consider the Hamiltonian equation:

$$\frac{dz}{d\theta} = X_{H_\theta}(z). \tag{0.1}$$

Let $\mathcal{P}(H)$ be the set of periodic-1 solutions of (0.1). Clearly $\mathcal{P}(H)$ is one to one correspondence to the set of fixed points of the time-1 flows ϕ_1^H of V associated to (0.1). For a "generic" choice of H, the graph $\Gamma_{\phi_1^H}$ of ϕ_1^H is transversal to the diagonal $\triangle_V$ in $V \times V$. It follows that $\mathcal{P}(H)$ is finite in this case. We refer this as a nondegenerate case. By the Lefschetz fixed point theorem, the algebraic cardinality of $\mathcal{P}(H)$ is just the Euler characteristic $\chi(V)$ of V, which is the alternating sum of the Betti number $b_i(V)$ of V. However, it has been conjectured by V.I. Arnold in [1] that the geometric cardinality of $\mathcal{P}(H)$ should satisfy a Morse inequality, $\#\mathcal{P}(H) \geq \sum_i b_i(V)$. This yields a much stronger estimate than what is expected by algebraic topology and reflects the remarkable symplectic rigidity (see [2] and [9]). This famous conjecture

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