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BLASCHKE-SANTALÓ INEQUALITIES

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Two of the most important affine isoperimetric inequalities are the Blaschke-Santaló inequality (see, e.g., Gardner [6, p. 322] or Schneider [12, p. 425]) and the classical affine isoperimetric inequality of affine differential geometry (see, e.g., Schneider [12, p. 419]). These two inequalities are closely related in that given either one of these inequalities, then by well-known methods one can be quickly deduced from the other. The aim of this article is to establish a new family of analytic inequalities and their geometric counterparts. One of the members of this family of inequalities turns out to be the Blaschke-Santaló inequality.

Let S^{n-1} denote the unit sphere in $\mathbb{R}^n$. Let B denote the unit ball (the convex hull of S^{n-1}) in $\mathbb{R}^n$, and write ω_n for the n-dimensional volume of B. Note that,

$$\omega_n = \pi^{n/2}/\Gamma(1 + \tfrac{n}{2}),$$

defines ω_n for all non–negative real n (not just the positive integers). For real $p \geq 1$, define $c_{n,p}$ by

$$c_{n,p} = \frac{\omega_{n+p}}{\omega_2 \omega_n \omega_{p-1}}.$$

For real $p \geq 1$ and continuous $f : S^{n-1} \to \mathbb{R}$, let $\|f\|_p$ denote the standard L_p-norm of f; i.e.,

$$\|f\|_p = \left\{ \int_{S^{n-1}} |f(u)|^p \, du \right\}^{1/p},$$

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