TOWARDS THE HOMOLOGY OF HURWITZ SPACES

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Abstract

The purpose of this paper is to construct a complex which computes the homology of Hurwitz spaces of branched covers of $\mathbb{P}^1$. We also compute some of the low dimensional homology groups of a compactification of the Hurwitz space, and report on computer calculations performed in specific examples.

1. Introduction

In this paper we develop a cell complex that computes the homology of the Hurwitz space of branched covers of $\mathbb{P}^1$. The motivation for this construction is to compute the Picard group of $SH_{k,b}$, the Hurwitz space parametrizing degree-k covers of $\mathbb{P}^1$ simply branched over b points. In particular, the second author conjectures that $Pic(SH_{k,b}) \otimes \mathbb{Q} = 0$, and using calculations made with our complex we present evidence for the validity of this conjecture.

Our complex also easily explains some known facts about the homology of $SH_{k,b}$, leading us to believe that the complex is natural and worthy of further investigation. For example, our complex comes from a representation of a compactification of $SH_{k,b}$ as a cell complex with no cells in codimension b or higher contained in $SH_{k,b}$. It follows that $H^i(SH_{k,b}) = 0$ for $i > b$, a fact which is also a consequence of $SH_{k,b}$ being affine. Likewise, calculating the top homology of our compactification easily reduces to the classical combinatorial problem associated