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THE KODAIRA DIMENSION OF THE MODULI SPACE OF CURVES OF GENUS 15

MEI-CHU CHANG & ZIV RAN

0. Introduction

The purpose of this paper is to prove that the moduli space $\mathscr{M}_{15}$ of curves of genus 15 has Kodaira dimension $\kappa = -\infty$, i.e. that $H^0(\mathscr{M}_{15}, nK_{\mathscr{M}_{15}}) = 0$ for all $n > 0$.

We recall briefly the current state of affairs regarding the structure of $\mathscr{M}_g$: $\mathscr{M}_g$ is unirational (in particular, has $\kappa = -\infty$) for $g \leqslant 13$ ([2], [4], [5], [13]), has $\kappa \geqslant 1$ for all $g \geqslant 23$, and is of general type for $g \geqslant 24$ ([6], [8], [9]).

Our proof is based on an analysis of a particular divisor $D \subset \overline{\mathscr{M}}_{15}$, namely

$$D = \text{some component of the locus of curves carrying a } g^3_{14}.$$

We show that D is unirational. Moreover, for some rational curve $F \subset D$ which is a member of a family of rational curves "filling up" D, we show that the intersection numbers

$$(*) \qquad D \cdot F > 0, \text{ while } F \cdot K_{\mathscr{M}_{15}} < 0.$$

As is easily seen, this implies $\kappa(\mathscr{M}_{15}) = -\infty$.

Our analysis of D is based on a correspondence

$$\mathscr{N} \leftarrow \rightarrow \mathscr{H} \dashrightarrow \mathscr{M}_{15},$$

where $\mathscr{H}$ is the closure of some component of the Hilbert scheme of nonsingular curves of genus 15 and degree 14 in $\mathbf{P}^3$, and $\mathscr{H} \dashrightarrow \mathscr{M}_{15}$ is the natural rational map. On the other hand, essentially, $\mathscr{N}$ = space of 4×9 matrices

$$A = \left(\boxed{\underbrace{L}_{8} \quad \underbrace{Q}_{1}} \right),$$

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