J. DIFFERENTIAL GEOMETRY
24 (1986) 53–68

RADON TRANSFORMS ON HIGHER RANK GRASSMANNIANS

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Abstract

We define a Radon transform R from functions $\mathrm{Gr}(k, n)$, the Grassmannian of projective k-planes in $\mathbf{C}P^n$ to functions on $\mathrm{Gr}(l, n)$. If $f \in C^\infty(\mathrm{Gr}(k, n))$ and $L \in \mathrm{Gr}(l, n)$, then $Rf(L)$ is the integral of $f(H)$ over all k-planes H which lie in L. If R' is the dual transform, we show under suitable assumptions on k and l that $R'R$ is invertible by a polynomial in the Casimir operators of $U(n+1)$, the group of isometries $\mathbf{C}P^n$. We also treat the real and quaternionic cases. Finally, we indicate some possible variations and generalizations to flag manifolds.

0. Introduction

Let P^n be a projective space over a real division ring, say $\mathbf{C}P^n$. The projective hyperplane transform, or *Radon transform* R, associates to a suitable function f on the projective space P^n a function Rf on P^{n*}, the space of projective hyperplanes in P^n, by integration: if H is a hyperplane in P^n, then

$$Rf(H) = \int_H f\,d\mu,$$

where $d\mu$ is normalized invariant measure. These transforms were first considered by S. Helgason. In [6] Helgason gave inversion formulas for R defined over real, complex, quaternionic, or octonionic projective spaces. A natural generalization of R is the k-plane transform R_k. This associates to f a function $R_k f$ on $\mathrm{Gr}(k, n)$, the Grassmann manifold of projective k-planes in P^n, again by integration. Helgason also gave inversion formulas for R_k defined

Received May 8, 1985 and, in revised form, September 24, 1985.
Key words and phrases. Grassmannians, projective spaces, Radon transforms, integral geometry, Casimir operators, Harmonic analysis, Penrose correspondence, flag manifolds.