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CORRESPONDENCE OF MODULAR FORMS TO CYCLES ASSOCIATED TO $O(p,q)$

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Introduction

In their study of Hilbert modular surfaces, Hirzebruch and Zagier [13] have discovered a striking connection between geometry and number theory. It was established that intersection numbers of cycles are Fourier coefficients of modular forms for Hilbert modular surfaces. Since then, the study of certain liftings of automorphic forms and their relation to geodesic cycles in quotients of symmetric spaces has been of great interest. The first subsequent big advance was made by Kudla and Millson [23] for their work on $SO(p,1)$ which offers a systematic and fruitful approach to the general case. They took the reductive pair $O(p,1) \times \mathrm{Sp}(2r, \mathbf{R})$ as their framework and used Weil representation to construct a theta function which has a geometric realization. Besides technical problems, they presented a feasible scheme for the general case.

In [33], [34], the analogous problems for $SU(p,1)$ were solved by Y. L. Tong and the author. In [35], we gave a correspondence, in the form of a geometric lifting, from Hermitian cusp forms of weight $p+2$ to certain harmonic differential forms of degree $(2,2)$ in compact quotients of $SU(p,2)$. This is the first example for symmetric spaces of higher rank. In [36], we returned to $SU(p,1)$ to discuss the case of noncompact quotients. In these studies, we witnessed tremendous technical complexity and gradually shifted our reliance on invariant theory. It should be mentioned here that we were inspired by Howe's recent effort [14]–[16] to emphasize the importance of classical invariant theory.

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