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BAND ASYMPTOTICS ON LINE BUNDLES OVER S^2

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1. Introduction

Let $E \to M$ be a Hermitian line bundle over a compact Riemannian manifold M. The choice of a connection, $\mathbf{a}$, on E which is compatible with the Hermitian structure determines a Bochner-Laplace operator, $\Delta_{\mathbf{a}}$, acting on the sections of E. If $q \in C^\infty(M)$, we can form the Schrödinger operator $\Delta_{\mathbf{a}} + q$ and consider its selfadjoint extension to $L^2(E)$. The objective of this paper is to study some spectral properties of this operator in the special case when M is the standard 2-sphere. This problem has been studied recently by Ruishi Kuwabara, [3], who showed that if the curvature of the connection is an odd 2-form the spectrum of the Schrödinger operator forms "bands" of fixed width about the eigenvalues of the Laplacian associated to the SO(3)-invariant connection. In this paper we sharpen Kuwabara's results and describe the asymptotic distribution of eigenvalues in the bands, thus generalizing a theorem of Weinstein's [5], in the flat case.

Added in proof. The referee has alerted us to another paper by Kuwabara which has appeared recently (Math. Z. **187** (1984) 481–490). In this paper asymptotic distributions of eigenvalues are obtained for connections in which the curvature is *not* odd (in which case there is no clustering). The result described in §4 can be regarded as a second order refinement of this result in the same sense that [1] is a second order refinement of [5]

2. Preliminaries

The set of isomorphism classes of line bundles over S^2 is indexed by the integers, the indexing map being the first Chern class followed by the canonical isomorphic $H^2(S^2, \mathbf{Z}) \cong \mathbf{Z}$. More precisely, for every $m \in \mathbf{Z}$ there is an essentially unique Hermitian line bundle with Chern number m, $E_m \to S^2$, and

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