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THE GAUSS MAP OF SURFACES IN $\mathbf{R}^n$

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1. Introduction

Let S be an oriented 2-dimensional surface immersed in a euclidean n-space $\mathbf{R}^n$. We denote by

$$g\colon S \to G_{2,n} \tag{1.1}$$

the generalized Gauss map, where $G_{2,n}$ is the Grassmannian of oriented 2-planes in $\mathbf{R}^n$, and for each point p of S, $g(p)$ is the tangent plane to S at p. Our object in this paper is to study properties of the map g, particularly those related to the geometry of S in $\mathbf{R}^n$ and the conformal structure of S.

The main problems we consider are:

1. Let S_0 be a Riemann surface, and

$$X\colon S_0 \to S \subset \mathbf{R}^n \tag{1.2}$$

a conformal immersion realizing S. What properties does the map

$$G = g \circ X\colon S_0 \to G_{2,n} \tag{1.3}$$

possesses by virtue of being defined via (1.1) as the Gauss map of a surface in $\mathbf{R}^n$?

2. Given a map

$$G\colon S_0 \to G_{2,n} \tag{1.4}$$

defined on a Riemann surface S_0, when does there exist a conformal immersion X of S_0 onto a surface S in $\mathbf{R}^n$ such that G is of the form (1.3), where g is the Gauss map of S?

3. To what extent is a surface S given by (1.2) determined by its Gauss map g?

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