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A CHARACTERIZATION OF THE 3RD STANDARD IMMERSIONS OF SPHERES INTO A SPHERE

HISAO NAKAGAWA

Dedicated to Professor Shigeru Ishihara on his 60*th birthday*

Introduction

Let M and $\overline{M}$ be complete connected Riemannian manifolds of dimension $n(\geqslant 2)$ and $n + p$ respectively. Hong [3] introduced a notion of planar geodesic immersions as follows: An isometric immersion f of M into $\overline{M}$ is called a *planar geodesic immersion* if each geodesic on M is locally mapped under the immersion into a 2-dimensional totally geodesic submanifold of $\overline{M}$. Planar geodesic immersions of M into an $(n + p)$-dimensional sphere $S^{n+p}(c)$ of constant curvature c have been completely classified by Little [4] and Sakamoto [9] independently, who stated that M is a compact symmetric space of rank one, and f is rigid to the 2nd or 1st standard immersion according as M is a sphere or not. In particular, concerning with isotropic immersions which are introduced by O'Neill [8], Sakamoto proves that the following properties are equivalent:

(1) f is nonzero constant isotropic and parallel,

(2) f is planar geodesic,

(3) for any geodesic γ on M, $f \circ \gamma$ is a circle on $\overline{M}$.

On the other hand, minimal immersions of compact symmetric spaces into a sphere have been investigated by Wallach [11]. Let $M = G/K$ be a compact symmetric space where the isotropy action of K is irreducible, and let Δ be the Laplacian operator for $(M, \langle\, ,\rangle)$, where $\langle\, ,\rangle$ is some G-invariant Riemannian structure up to scalar multiple. Let V_λ be an eigenspace with an eigenvalue λ of Δ, and for any real-valued functions g_1 and g_2 on M, let $(g_1, g_2) = \int_M g_1 g_2 dM$. Then V_λ is a vector space over $\mathbf{R}$ endowed with the inner product $(\, ,)$. For each nonzero eigenvalue λ, let $\{g_1, \cdots, g_{q+1}\}$ be an orthonormal basis of V_λ, where

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