J. DIFFERENTIAL GEOMETRY
12 (1977) 619-627

THE SECOND FUNDAMENTAL FORM OF A PLANE FIELD

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This paper arose out of attempts to understand geometrically the meaning of various foliation invariants introduced in the last few years. Because these invariants are associated to the normal bundle, the characteristics of the normal plane field are important. Since the normal plane field is not integrable, one is led to study the Riemannian geometry of arbitrary plane fields, which is done by generalizing the second fundamental form. Concepts such as mean curvature and minimality can then be introduced for a plane field, and it can be shown that a totally geodesic plane field has vanishing second fundamental form. This is of interest because the normal plane field to an R-foliation is totally geodesic.

Given a foliation of a Riemannian manifold, a foliation connection is chosen in the normal bundle which is as compatible as possible with the Riemannian connection. Certain formulas are developed for the components of the connection and curvature forms, then used to prove a number of results, including: the leaf classes h_i for odd i depend only on the second fundamental form of the normal plane field, the Godbillon-Vey class in higher codimension is given by a formula analogous to that of Reinhart and Wood [8], and the reductions modulo the integers of certain leaf classes of dimension $4j - 1$ are independent of the choice of framing (they are defined only for framed foliations). A method for calculating the cohomology of truncated relative Weil algebras is essential to obtain the results. Such a method has been given in general by Kamber and Tondeur [9], [10], while more recently Guelorget and Joubert [5], building on their work, have given very explicit formulas for the case needed here, the general linear algebra modulo the orthogonal algebra.

Finally, some examples are given of vector fields in euclidean 3-space such that the normal plane fields are not integrable, and their second fundamental forms have certain prescribed properties.

1. Plane fields

A smooth p-plane field on a smooth Riemannian n-manifold is assumed. The inner product will be symbolized by $\langle\ ,\ \rangle$, while the Riemannian covari-

Received December 13, 1975, and, in revised form, August 25, 1976. Research supported by the National Science Foundation under grant MPS 73-08938A02.