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ESTIMATES OF THE LENGTH OF A CURVE

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In this article we establish some upper bounds for the length of a curve γ lying in a convex region T of an n-dimensional Riemannian space. The results obtained here have the character of a comparison theorem of the following type. Let k_s, κ be respectively the minimum values of the sectional curvature in T and of the normal curvature of the boundary of T. Under the condition that $k_s > -\kappa^2$, one can assign to the region T a circle T_0 in a k_s-plane (a two-dimensional sphere, plane or hyperbolic plane of curvature k_s) whose boundary has the geodesic curvature κ. Then, if the maximum curvature ξ of γ is less than κ, the length of γ does not exceed the length of the longest arc contained in T_0, having constant curvature ξ. (See the corollary of Theorem 1 of § 1.)

The question on estimates of the length of a curve a in a region on a two-dimensional surface was explored by A. D. Aleksandrov and V. V. Strel'cov in 1953 (see [1]). The estimates obtained in [1] contain some integral characteristics of the curve and the region. Their estimates and ours (when $n = 2$) do not follow from one another.

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1. The basic construction and the results

In n-dimensional Riemannian space M, $n \geq 2$ (of regularity class C^4) we consider a connected region which has a compact closure T and is bounded by a nonempty, possibly disconnected regular hypersurface Γ (of class C^4). The surface Γ divides a sufficiently small ball neighborhood of any of its points into two components; we suppose that only one of them belongs to T. (Instead of this we could suppose that T is the image under an immersion of some connected compact n-dimensional manifold with a smooth edge into M.) Let the boundary Γ of the region T be strictly convex in the following sense: all the normal curvatures of Γ on the side of the interior normal are not less than some positive κ. Finally, let us suppose that in the compact region T the

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