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VARIATIONAL PROPERTIES OF FUNCTIONS OF THE MEAN CURVATURES FOR HYPERSURFACES IN SPACE FORMS

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Dedicated to the author's father on his 70th birthday

Introduction

The formula for the first variation, with fixed boundary, of the volume integral for a hypersurface M^n in Euclidean $(n+1)$-space E^{n+1} is well-known. It is, [4, p. 178], $\delta \int_M 1 \cdot dV = -n \int_M \sigma_1 (N \cdot \xi) dV$, where σ_1 is the first mean curvature, N is the unit normal, and ξ is the deformation vector. Recently this classical formula has been generalized by several mathematicians including Pinl and Trapp [10] and the author [12]. In [12] we show that if σ_r, $r = 0, 1, \cdots, n$, denotes the r-th mean curvature function, then $\delta \int_M \sigma_r dV = -(n-r) \cdot \int_M \sigma_{r+1}(N, \xi) dV$. This is shown in [10] when $r = 1$ or n. We prove similar formulas in [12] for submanifolds of arbitrary codimension when r is even. The results of [10] and [12] for hypersurfaces are proved by Rund [13] in a more general setting. The object of the present paper is to study the variation of $\int_M f(S_1, \cdots, S_n) dV$, where M is a hypersurface in a space form $N^{n+1}(c)$ of curvature c, $S_r = C_r^n \sigma_r$ is the r-th elementary symmetric function of the principal curvatures (C_r^n being the binomial coefficient), and f is any smooth function. If $c = 0$, we also consider $\int_M f(S_1, \cdots, S_n, P, Q) dV$, where P is the support function, and $2Q$ is the square of the length of the position vector. Many of our results could be derived from the theory in [13] but it appears that because we study a less general case here our methods are more elementary than those of [13].

We begin by deriving the formula for the first variation of our integral as well as the formula for the second variation in those cases (see above) studied

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