

MODULI AND MODULAR GROUPS OF A CLASS OF CALABI-YAU n -DIMENSIONAL MANIFOLDS, $n \geq 3$

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1. Introduction

Since the discovery of mirror symmetry in string theory by physicists, there have been tremendous activities on Calabi-Yau manifolds both by physicists and mathematicians. The reason that mirror symmetry has attracted a lot of mathematicians' attention is that it predicts successfully the number n_k of rational curves of degree k in these manifolds. This so-called Mirror Conjecture was first solved recently by Lian, Liu and Yau in their celebrated work [3]. In this paper we shall study the geometry of distinguished class of Calabi-Yau manifolds

(1.1)

$$X_s = \{(x_1 : \cdots : x_n) \in \mathbf{CP}^{n-1} : x_1^n + \cdots + x_n^n + sx_1x_2 \cdots x_n = 0\}.$$

For $n = 5$, this class of Calabi-Yau 3-manifolds were studied in detail by Candelas, Ossa, Green and Parkers [1] by means of the period map. In particular, they observed that the modular group is not $SL(2, \mathbf{Z})$.

It is the purpose of this paper to find out the moduli and the modular group of this one-parameter family of Calabi-Yau manifolds in (1.1) for all $n \geq 5$. Our argument is uniform for all $n \geq 5$. We remark that $n = 3$ was treated by our previous paper [2] with different motivation. The crucial contribution of our paper is the introduction of some special points in Calabi-Yau manifolds.

Let ρ_i , $i = 1, 2, \dots, n$, be n -distinct roots of $x^n = -1$. It is clear that the following $N = \frac{1}{2}n^2(n-1)$ points Q_1, \dots, Q_N of the form

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