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Weak-* Maximality of Certain Hardy Algebras $H^{\infty}(m)$

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The purpose of this paper is to discuss the weak-*maximality of certain Hardy algebras $H^{\infty}(m)$. Merrill [7] obtained conditions for the maximality of Hardy algebras for logmodular algebras. In this paper we study this problem for hypo-Dirichlet algebras and obtain a similar result as one of Merrill. We also discuss as an application the(uniform) maximality of certain classes of hypo-Dirichlet algebras.

§1. Preliminaries.

Let A be a uniform algebra on a compact Hausdorff space X, i.e., let A be a closed subalgebra in C(X) separating points in X and containing constant functions on X, where C(X) denotes the Banach algebra of complex-valued continuous functions on X with the supremum norm. A is called a hypo-Dirichlet algebra on X if there exist finite elements $Z_1, Z_2, \dots, Z_{\sigma}$ in the family A^{-1} of invertible elements of A such that the real linear space of functions of the form of

$$\operatorname{Re}(f) + \sum_{i=1}^{\sigma} c_i \log |Z_i| \quad (f \in A, c_i \in \mathbb{R})$$

is dense in the space $C_{\mathbb{R}}(X)$ of real continuous functions on X.

Now let A be a hypo-Dirichlet algebra and M_A be the maximal ideal space of A. Then each element ϕ of M_A has a finite dimensional set M_{ϕ} of representing measures on X for ϕ . And every $\phi \in M_A$ has a unique Arens-Singer measure m on X. A positive measure m on X is called an Arens-Singer measure for ϕ if $\log |\phi(f)| = \int \log |f| dm$ for all $f \in A^{-1}$ ([1]; [4], p. 116).

The abstract Hardy spaces $H^{p}(m)$, $1 \leq p \leq \infty$, associated with A are defined as follows; for $1 \leq p < \infty$, $H^{p}(m)$ is the $L^{p}(m)$ -closure of A and $H^{\infty}(m)$ is the weak-*closure of A in $L^{\infty}(m)$. We see that $H^{\infty}(m)$ is an

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