TOKYO J. MATH.
VOL. 1, No. 1, 1978

On Bounded Solutions of $x''=t^{\beta}x^{1+\alpha}$

Tosiya SAITO

Keio University

§1. In this paper, we consider a second order nonlinear differential equation

$$(1) \qquad x''=t^{\beta}x^{1+\alpha} \qquad ('=d/dt)$$

where α and β are real numbers and $\alpha>0$. This equation includes, as its special case, the equation

$$x''=t^{1-m}x^{m}\,, \qquad 1<m<3\,,$$

which is known as Emden's equation [1].

The solutions of (1) considered here are those which assume real values for real t. Therefore, for any given α and β, t^{β} and $x^{1+\alpha}$ must be regarded as representing real-valued branches. So it is quite natural to assume that

(1) the domain in which the equation (1) is considered is

$$G\colon 0<t<\infty\,, \qquad 0\leqq x<\infty\,,$$

(2) $x^{1+\alpha}$ and t^{β} represent their nonnegative-valued branches in G.

The purpose of the present paper is to show that the equation (1) has a one-parameter family of (positive) bounded solutions if β satisfies a certain condition. Here, by a bounded solution, we mean a solution $x(t)$ such that $x(t)$ and $x'(t)$ are both bounded for $0<t<\infty$.

§2. Let $x(t)$ be a bounded solution of (1). Since

$$x''(t)=t^{\beta}(x(t))^{1+\alpha}\geqq 0$$

in G by our assumptions given at the outset, $x'(t)$ is a nondecreasing function of t. So if $x'(a)>0$ for some $a>0$, we have

$$x'(t)\geqq x'(a) \qquad \text{for} \quad t\geqq a\,.$$

Received March 1, 1978