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Classification of Finitely Determined Singularities of Formal Vector Fields on the Plane

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Introduction

The author gave a characterization of finite determinacy of formal vector fields in [6, 7]. Originally the problem of finite determinacy was proposed by R. Thom [8] for C^∞-map germs and J. Mather gave a complete answer to it [10]. Specially, for C^∞-functions this concept is very important in connection with the elementary catastroph theory [16]. Roughly speaking, for C^∞-functions k-determinacy has upper semi-continuity on k and the local structure of the orbit decomposition of function space by the action of the local diffeomorphisms is reduced to that of finite jet space.

On the other hand, for vector fields the situation is quite different. Upper semi-continuity of k-determinacy is lost and the local orbit decomposition is not reduced to that of finite jet space. Moreover, even local triviality of orbits does not hold. Thus we can not hope to construct an unfolding theory for vector fields except for some exceptional cases. However, in this paper we see that in 2-dimensional case the classification and the hierarchy can be simply described for finitely determined singularities of formal vector fields.

§1. Definitions and the results.

Let C be the field of complex numbers. Let $\mathscr{F}=C[[x, y]]$ be the formal power series algebra. We denote by $\mathfrak{X}^0$ the set of formal vector fields (i.e. derivations of $\mathscr{F}$) which have no constant terms. Naturally $\mathfrak{X}^0$ has Lie algebra structure and we denote by $[\ ,\ ]$ its Lie bracket. Let G be the group of algebra automorphisms of $\mathscr{F}$. The group G acts on $\mathfrak{X}^0$ as $\varphi_* X=\varphi^{-1}X\varphi$ where $\varphi\in G$ and $X\in\mathfrak{X}^0$. We say that two formal vector fields X and Y are *equivalent* if there is an element $\varphi\in G$ such

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