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A New Characterization of Dragon and Dynamical System

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Introduction

The fractal sets called a twindragon and a dragon are encountered in a complex binary representation [7] and a paper folding curve [5], respectively. We have constructed in a previous paper [1] dynamical systems on the twindragon (Figure 1) and the tetradragon (Figure 2) tiled by four dragons which are obtained as realized domains for a two state Bernoulli shift and a some subshift with a finite coding from a Markov subshift [8], respectively.

We propose in this paper a new construction of a dragon different from the paper folding process and consider a dynamical system on a domain, tiled by four dragons, which are not the tetradragon. We call this domain a cross dragon. Moreover surprisingly we can show in Section 4 that this cross dragon system is actually a dual system [1] of a very simple group endomorphism.

Indeed the cross dragon system is obtained as a realization of a following Markov subshift. Let $M=(M_{j,k})$, $1\leqq j, k\leqq 4$, be a matrix such that

$$M=\begin{pmatrix}0 & 1 & 1 & 0\\ 0 & 0 & 1 & 1\\ 1 & 0 & 0 & 1\\ 1 & 1 & 0 & 0\end{pmatrix}.$$

We consider M as a structure matrix for a state space $\Gamma=\{0, i, -1+i, -1\}$ by a correspondence $\tau: \{1, 2, 3, 4\}\to\Gamma$ such that $\tau[1]=0$, $\tau[2]=i$, $\tau[3]=-1+i$ and $\tau[4]=-1$, that is, let V be a set of infinite sequences generated by the structure matrix M,

$$V=\{(\gamma_1, \gamma_2, \cdots); M_{\gamma_j,\gamma_{j+1}}=1,\ \gamma_j\in\Gamma \text{ for all } j\in N\},$$

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