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On the Number of Cusps of Stable Perturbations of a Plane-to-Plane Singularity

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Introduction

Let $f: (\boldsymbol{R}^2, O) \to (\boldsymbol{R}^2, O)$ be a smooth map germ. By the theorem of Whitney [Wh], f can be approximated (in the semi-local sense) by a C^∞ stable mapping. In other words, f is a "degeneration" of neighboring stable mappings, which we call stable perturbations of f. Then it will be natural to expect that stable perturbations have several properties in common reflecting the structure of the "generating" map-germ f.

In this paper we concentrate on investigating the number of cusps of stable perturbations of a generic plane-to-plane singularity. For instance, we observe that the number $\kappa(\tilde{f})$ modulo 2 of cusps of a stable perturbation $\tilde{f}$ of a generic map-germ $f: (\boldsymbol{R}^2, O) \to (\boldsymbol{R}^2, O)$ is a topological invariant of f (Theorem 2.4). In fact $\kappa(\tilde{f}) \bmod 2$ is determined by the number of branches of the locus of critical points of f and the mapping degree of f (Theorem 2.1). Thus if two generic map-germs $f, g: (\boldsymbol{R}^2, O) \to (\boldsymbol{R}^2, O)$ are topologically equivalent, then the parities of $\kappa(\tilde{f})$ and $\kappa(\tilde{g})$ are coincident for any stable perturbations $\tilde{f}$ of f and $\tilde{g}$ of g.

This observation is obtained as an application of a global formula for singularities of maps between oriented 2-manifolds with boundary (Theorem 1.1), which is a modified form of Quine's formula [Q]. The topological invariant $\kappa(\tilde{f}) \bmod 2$ is algebraically calculable from f (Theorem 2.2).

In §1, our global formula is proved from Quine's formula. In §2, the genericity condition is explained and $\kappa(\tilde{f}) \bmod 2$ is investigated for stable perturbations $\tilde{f}$ of a generic map-germ $f: (\boldsymbol{R}^2, O) \to (\boldsymbol{R}^2, O)$. Another restriction for the number $\kappa(f_t)$ of cusps near the origin for a deformation $\{f_t\}$ of f is obtained in §3, using complex analytic geometry.

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