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2-Ended Manifolds and Locally Splitting Actions of Abelian Groups

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The purpose of this paper is to prove the following result.

THEOREM. *Let (S^1, M) be a locally splitting action and suppose that the universal covering manifold $\tilde{M}$ is 2-ended. Then, every isometry of M preserves the foliated structure defined by the action.*

In the sequel, we apply some consequences of this result in order to study the structure of the identity component $I_0(M)$, of the isometry group $I(M)$ of the Riemannian manifold M.

1. Introduction.

The closed connected subgroup G of $I_0(M)$ acts *locally splitting* if and only if it has only one orbit type, the normal to the orbits distribution $\mathfrak{N}$ is integrable and every fundamental (Killing) vector field has constant length in directions normal to the orbits of the action [1].

For G abelian, it turns out that a locally splitting action of G is *free*, it has *parallel* fundamental vector fields and $\tilde{M}$ is a Riemannian product $\boldsymbol{R}^k \times \tilde{W}$, $k = \dim G$ [4, Theorem B]. In particular, if $G = S^1$ and $\tilde{M}$ has two *ends* then $\tilde{W}$ is compact and the orbits of the *lifted* in $\tilde{M}$ action (cf. [2] for the used terminology) admit a parametrization which makes them *lines* (see [6]). With this notation the motivation of the present work is twofold.

It is always interesting to decide when a Killing vector field of M preserves some foliated structure ([3], [5]). As every locally splitting action defines a local product structure consisting of orbits and integral manifolds, it seems natural to discuss similar problems in our case too. Our main result shows that, not only flows of Killing vector fields but, *every* isometry maps orbits of the given action to orbits and interchanges the maximal integral manifolds $N(x)$, $x \in M$, of $\mathfrak{N}$.

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