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Determinant Surfaces of Rank 2 Bundles on P^3

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§1. Introduction.

The aim of this paper is to study the relationship between stable vector bundles $\mathscr{E}$ of rank two on P^3 and their determinant surfaces S determined by two sections of $\mathscr{E}$. We discuss specifically the case $c_1(\mathscr{E})=4$ in detail.

Vector bundles on a variety are closely related to its special subvarieties. On P^3, a general surface has Picard number one by the Noether-Lefschetz theorem (cf. [Lo]): If S is a general surface of degree $d\geq 4$ in P^3, then $\operatorname{Pic} S\cong Z$ with the generator $\mathscr{O}_S(1)$. On the other hand, a smooth determinant surface S is not general because its Picard number is at least two by Theorem 3.1:

THEOREM 1.1. *A smooth surface S in P^3 occurs as a determinant surface of a rank two vector bundle $\mathscr{E}$ on P^3 if and only if S has a surjective morphism onto P^1.*

In this paper we give an estimate of $\rho(S)$ from below in terms of the behaviour of $\mathscr{E}$ under the restriction to lines and planes. Defining the jumping planes in (5.5), we can state a sufficient condition for S to have Picard number ≥ 3 in (5.6). Moreover we have the following estimate:

THEOREM 1.2. *Let $\mathscr{E}$ be a stable vector bundle of rank two on P^3 with $c_1(\mathscr{E})=4$ and $c_2(\mathscr{E})\geq 9$. Suppose that $\mathscr{E}$ has a smooth determinant surface S and that $c_2(\mathscr{E})/(h^1(\mathscr{E}(-4))+1)=$(degree of a fibre of the Stein factorization of the morphism $S\to P^1$ as in Theorem 1.1)≥ 4. Then*

$$\rho(S)\geq 2+\tfrac{1}{2}\# J(\mathscr{E}),$$

where $\# J(\mathscr{E})$ is the number of jumping planes for $\mathscr{E}$.

As a corollary of these theorems and (2.13), we have:

COROLLARY 1.3. *For any given $c_2\geq 5$, there exists a stable vector bundle $\mathscr{E}$ of rank*

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