

Very Special Framed Links for a Homotopy 3-Sphere

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Introduction.

A *framed link* K in a closed 3-manifold M is a system of disjoint simple closed curves $K = (\kappa_1, \kappa_2, \dots, \kappa_n)$ equipped with another system of simple closed curves $\tilde{K} = (\tilde{\kappa}_1, \tilde{\kappa}_2, \dots, \tilde{\kappa}_n)$ such that each component $\tilde{\kappa}_j$, called the *framing curve* of κ_j , lies on the boundary of a regular neighborhood V_j of κ_j in M and meets the meridian curve of V_j exactly once. Given a framed link K in M , a 3-manifold $\chi(M; K)$ obtained by a *Dehn surgery* along K is defined as follows:

$$\chi(M; K) = (M - (\overset{\circ}{V}_1 \cup \overset{\circ}{V}_2 \cup \dots \cup \overset{\circ}{V}_n)) \cup (V'_1 \cup V'_2 \cup \dots \cup V'_n),$$

where each V'_j is a solid torus glued back by a homeomorphism $h_j : \partial V'_j \rightarrow \partial V_j$ which takes a meridian curve of V'_j onto the framing curve $\tilde{\kappa}_j$. Furthermore we can define the *dual framed link* $K^* = (\kappa_1^*, \kappa_2^*, \dots, \kappa_n^*)$ in $M^* \equiv \chi(M; K)$ so that each κ_j^* is a core of V'_j and its framing curve $\tilde{\kappa}_j^*$ is a meridian of V_j , and we get the dual surgery description $M = \chi(M^*; K^*)$.

In [1] it was shown that any closed 3-manifold M has a framed link K such that $\chi(M; K)$ is homeomorphic to a 3-sphere S^3 and the dual framed link K^* in S^3 enjoys some special properties, especially K^* forms a pure plat in S^3 . It is pointed out in [1] that such a framed link of M is closely related to a Heegaard splitting (or diagram) of M . On the other hand, using a notion of a d -pseudo core, we proposed in [2] a condition for a Heegaard splitting to be reduced.

In this paper, to see how extent we can apply the conditions in [2] to a Heegaard splitting of a homotopy 3-sphere M induced by a surgery description, we will try to add further good properties to a special framed link given in [1]. In the case where $\chi(M; K) = S^3$ and the dual framed link K^* forms a pure plat, it was shown in [2] that we can take a link isotopic to K as a *generalized core* (see [2] for the definition) of the Heegaard splitting of M induced by the surgery description, and that a key for applying the reducibility condition is to find a *localizing arc system* (see [2] and §1.3 below) for a generalized core. In §1, adding some conditions on a localizing arc system to a special framed link defined in [1], we introduce a notion of a “very special framed link” of a closed 3-manifold, and in §§2, 3 we will prove that