TOKYO J. MATH.
VOL. 30, NO. 2, 2007

The Decomposability of Z_2-Manifolds in Cut-and-Paste Equivalence

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(Communicated by T. Kawasaki)

Introduction

All manifolds considered here are unoriented compact smooth manifolds with or without boundary. G denotes a finite abelian group, and G-manifolds mean manifolds with smooth G-action.

Let $m \geq 0$ be an integer. Let P and Q be m-dimensional compact G-manifolds with boundary, and $\varphi : \partial P \to \partial Q$ be a G-diffeomorphism. Pasting P and Q along the boundary by φ, we obtain a closed G-manifold $P \cup_\varphi Q$ after rounding a corner. If $\psi : \partial P \to \partial Q$ is a second G-diffeomorphism, we obtain a second closed G-manifold $P \cup_\psi Q$. The two closed G-manifolds $P \cup_\varphi Q$ and $P \cup_\psi Q$ are said to be *obtained from each other by cutting and pasting* (*S*chneiden und *K*leben in German). Two m-dimensional closed G-manifolds M and N are said to be *cut-and-paste equivalent*, or *SK-equivalent* to each other, if there is an m-dimensional closed G-manifold L such that the disjoint union $M + L$ is obtained from $N + L$ by a finite sequence of cuttings and pastings. This is an equivalence relation on $\mathfrak{M}_m^G$, the set of m-dimensional closed G-manifolds. Denote by $[M]$ the equivalence class represented by M, and by $\mathfrak{M}_m^G/SK$ the quotient set of $\mathfrak{M}_m^G$ by the SK-equivalence. $\mathfrak{M}_m^G/SK$ becomes a semigroup with the addition induced from the disjoint union of G-manifolds. The Grothendieck group of $\mathfrak{M}_m^G/SK$ is called the *SK-group* of m-dimensional closed G-manifolds and is denoted by SK_m^G. The direct sum $SK_*^G = \bigoplus_{m \geq 0} SK_m^G$ becomes a graded ring with multiplication induced from cartesian product, with diagonal G-action, of G-manifolds.

In Komiya [13] we dealt with the case in which G is of odd order, and obtained a necessary and sufficient condition for that, for a given $u \in SK_m^G$ and an integer $t \geq 0$, u is divisible by t, i.e., $u = tv$ for some $v \in SK_m^G$.

In the present paper we will deal with the case of $G = \mathbf{Z}_2$, the cyclic group of order 2. Using a result in Komiya [12], we will obtain a condition for a closed $\mathbf{Z}_2$-manifold M to decompose in the sense of SK-equivalence into the product $N \times L$ of two closed $\mathbf{Z}_2$-manifolds N and L. In fact, for given $u \in SK_m^{\mathbf{Z}_2}$ and $v \in SK_n^{\mathbf{Z}_2}$ with $n \leq m$, we will obtain a necessary

Received December 1, 2005; revised March 2, 2006