54. Gauss Decomposition of Connection Matrices and Application to Yang-Baxter Equation. I

By Kazuhiko AOMOTO*) and Yoshifumi KATO**)

(Communicated by Heisuke HIRONAKA, M. J. A., Sept. 13, 1993)

1. General scheme. Method of Gauss decomposition. Suppose that an $N \times N$ matrix $G = ((g_{i,j}))_{i,j=1}^{N}$ is given such that all the entries $g_{i,j}$ are functions of $x = (x_1, \ldots, x_m) \in (C^*)^m$. Let $\mathfrak{S}_m$ be the symmetric group of m th degree with the canonical generators $\tau_1, \ldots, \tau_{m-1}$, τ_j being the tranposition between the arguments x_j and x_{j+1}. Then we have the Coxeter relations, $\tau_j^2 = e$, $\tau_j \tau_{j+1} \tau_j = \tau_{j+1} \tau_j \tau_{j+1}$ (e denotes the identity). Let $S : \mathfrak{S}_m \ni \tau \to S_\tau \in GL_N(C)$ be a linear representation of $\mathfrak{S}_m$, i.e., $S_{\tau\tau'} = S_\tau S_{\tau'}$ and $S_e = 1$ and $\tau, \tau' \in \mathfrak{S}_m$.

We firstly assume the following property for $G(x)$.

(1.1) $$\tau G(x) \text{ (defined as } G(\tau^{-1}(x))) \mp S_\tau^{-1} \cdot G(x) \cdot S_\tau$$

for an arbitrary $\tau \in \mathfrak{S}_m$.

Let $G(x) = \Omega^*(x)^{-1} \cdot \Omega(x)$ be a Gauss decomposition of $G(x)$ such that $\Omega(x)$ is a lower triangular matrix and $\Omega^*(x)$ is an upper triangular one. Let the one cocycle $\{W_\tau(x)\}_{\tau \in \mathfrak{S}_m}$ with values in $GL_N(C)$ be defined as $W_\tau(x) = \Omega(x) \cdot S_\tau \cdot (\tau\Omega(x))^{-1} = \Omega^*(x) \cdot S_\tau \cdot (\tau\Omega^*(x))^{-1}$ so that we have

(1.2) $$W_{\tau\tau'}(x) = W_\tau(x) \cdot \tau W_{\tau'}(x) \text{ and } W_e(x) = 1.$$

Secondly we assume that each $W_{\tau_r}(x)$ depends only on x_{r+1}/x_r, i.e.,

(1.3) $$W_{\tau_r}(x) = W_r(x_{r+1}/x_r), \quad 1 \le r \le m-1.$$

The equalities $W_{\tau_j\tau_{j+1}\tau_j} = W_{\tau_{j+1}\tau_j\tau_{j+1}}$ and $W_{\tau_j^2} = 1$ and the cocycle condition shows the Yang-Baxter equation

(1.4) $$W_j(u)\, W_{j+1}(uv)\, W_j(v) = W_{j+1}(v)\, W_j(uv)\, W_{j+1}(u)$$

and

(1.5) $$W_j(u)\ W_j(u^{-1}) = 1.$$

Then we call the matrix $G(x)$ *admissible*. Admissible matrices appear in a natural manner as connection coefficients among the symmetric A-type Jackson integrals. In this note we shall state their explicit formulae without proof.

2. Symmetric A-type Jackson integrals and the associated principal connection matrices. Let $q \in C$, $|q| < 1$, be the elliptic modulus. We consider the symmetric A-type q-multiplicative function $\Phi_{n,m}(t)$ of $t = (t_1, \ldots, t_n)$ on the n-dimensional algebraic torus $(C^*)^n$,

(2.1) $$\Phi_{n,m}(t) = \Phi_{n,m}(t \mid x ; \alpha_1, \beta, \gamma)$$
$$= \Pi_{j=1}^n \left\{ t_j^{\alpha_j}\, \Pi_{k=1}^m \frac{(t_j/x_k)_\infty}{(t_j q^\beta / x_k)_\infty} \right\} \Pi_{1 \le i < j \le n} \frac{(q^{\gamma'} t_j / t_i)_\infty}{(q^\gamma t_j / t_i)_\infty}$$

*) Department of Mathematics, Nagoya University.
**) Department of Mathematics, Meijo University.