28. On Total Risk Aversion and Differential Games for Controlled Parabolic Equations

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1. Recently E. N. Barron and R. Jensen [1] investigated a connection between the theory of risk for controlled finite dimensional state systems and the theory of differential games. In the present note we will discuss the same problems for infinite dimensional state systems on a finite time interval $[0, T]$ governed by parabolic equations.

Let W be a standard d-dimensional Wiener process on a probability space $(\Omega, \mathcal{F}, P)$ and denote by $\mathcal{F}_t$ the σ-field generated by $\{W(s) ; s \leq t\}$. Let Γ be a compact subset of $\boldsymbol{R}^q$ and $\mathcal{A}$ be the space of all Borel measurable function $u : \boldsymbol{R}^n \to \Gamma$ endowed with the $L^2(\boldsymbol{R}^n \to \boldsymbol{R}^q ; e^{-|x|} dx)$-topology, called a control region. A map $U : [0, T] \times \Omega \to \mathcal{A}$ is called an admissible control, if it is $\mathcal{F}_t$-progressively measurable.

Putting $H = L^2(\boldsymbol{R}^n)$ and

$$A\zeta = \sum_{ij=1}^n \frac{\partial}{\partial x_i} (a^{ij}(x) \frac{\partial}{\partial x_j} \zeta) + \sum_{i=1}^n r^i(x) \frac{\partial}{\partial x_i} \zeta - c(x)\zeta,$$

we consider the controlled system ξ governed by the parabolic equation in a random environment:

(1) $\dfrac{\partial \xi}{\partial t}(t, x) = A\xi(t, x) + b(x, \xi(t, x), y + W(t), U(t, x)),$ for $t \in (0, T)$, $x \in \boldsymbol{R}^n$ with initial condition $\xi(0, \ ) = \eta (\in H)$.

Let us assume the following conditions (A1) ~(A5).

(A1) a^{ij} and r^i are in $C^3(\boldsymbol{R}^n)$, with finite C^3-norm,

(A2) the matrix $(a^{ij}(x))$ is uniformly positive difinite, say $(a^{ij}(x)) \geq \hat{\mu} I$ where $I = n \times n$ identity matrix and $\hat{\mu} > 0$,

(A3) c is bounded, continuous and non-negative,

(A4) $b ; \boldsymbol{R}^n \times \boldsymbol{R}^1 \times \boldsymbol{R}^d \times \Gamma \to \boldsymbol{R}^1$ is bounded and Lipschitz continuous,

(A5) there is $\hat{b} \in H$, such that $\hat{b}(x)$ is decreasing, as $|x| \to \infty$, and $|b(x, a, y, u)| \leq \hat{b}(x)$ for any a, y and u.

By (A2), $-A$ is coercive, namely there is a non-negative constant μ, such that

(A2)′ $\langle -A\zeta, \zeta\rangle + \mu \|\zeta\|^2 \geq 0$ for $\zeta \in H^1$,

where $\langle \, , \, \rangle$ = duality pairing between H^{-1} and H^1.

Define $\beta ; H \times \boldsymbol{R}^d \times \mathcal{A} \to H$ by $\beta(\zeta, y, u)(x) = b(x, \zeta(x), y, u(x))$. Let us put $m(a)^2 = \displaystyle\int \min (|a(x)|^2, \hat{b}(x)^2) dx$. Then (A4) implies

(2) $\|\beta(\zeta, y, u) - \beta(\zeta', y', u')\| \leq k(\|\zeta - \zeta'\| + m(y - y') + m(u - u'))$

with a constant k. In view of the fact that A generates a continuous semi-