

16. A Subadjunction Formula and Moishezon Fourfolds Homeomorphic to P_c^4

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§0. Introduction. The purpose of the present paper is to report some partial solutions to the following conjectures. Details [5] will appear elsewhere.

Conjecture MP_n . Any Moishezon complex manifold homeomorphic to P_c^n is isomorphic to P_c^n .

Conjecture DP_n . Any complex analytic (global) deformation of P_c^n is isomorphic to P_c^n .

Conjecture MP_n has been settled by Hirzebruch-Kodaira [1] and Yau [10] when the manifold under consideration is *projective or Kählerian*.

Recently Kollár [2] and the author [3] solved (MP_3) in the affirmative, each supplementing the other. Peternell [6] [7] also asserts (MP_3) .

(0.1) **Theorem [2] [3].** Any Moishezon threefold homeomorphic to P_c^3 is isomorphic to P_c^3 .

(0.2) **Theorem.** Let X be a Moishezon manifold of dimension n . Assume that there is a line bundle L on X such that $c_1(X) = dc_1(L)$ ($d \geq n+1$), $h^0(X, O_X(L)) \geq n+1$, and $\kappa(L) = n$. If a complete intersection of general $(n-1)$ -members of the complete linear system $|L|$ is nonempty outside the base locus $Bs |L|$, then X is isomorphic to P_c^n .

(0.3) **Theorem.** Let X be a Moishezon manifold homeomorphic to P_c^n , and L a line bundle on X with $L^n = 1$. Assume $h^0(X, O_X(L)) \geq n+1$. If a complete intersection of general $(n-1)$ -members of $|L|$ is nonempty outside $Bs |L|$, then X is isomorphic to P_c^n .

(0.4) **Theorem.** Let X be a Moishezon fourfold, and L a line bundle on X . Assume that $\text{Pic } X = \mathbb{Z}L$, $c_1(X) = dc_1(L)$ ($d \geq 5$) and $h^0(X, O_X(L)) \geq 5$. Then X is isomorphic to P_c^4 .

(0.5) **Theorem.** Let X be a Moishezon fourfold homeomorphic to P_c^4 , and L a line bundle on X with $L^4 = 1$. Assume $h^0(X, O_X(L)) \geq 3$. Then X is isomorphic to P_c^4 .

(0.6) **Corollary.** Any complex analytic (global) deformation of P_c^4 is isomorphic to P_c^4 .

§1. A complete intersection l and a subadjunction formula. (1.1) Let X be a compact complex manifold of dimension n , a line bundle L on X with $h^0(X, O_X(L)) \geq n-1$. Let V be a linear subspace of $H^0(X, L)$ of dimension $n-1$, $l := l_V$ a scheme-theoretic complete intersection associated with V . More precisely, the ideal sheaf of O_X defining l is given by $I_l = \sum_{s \in V} s O_X$.