12. *Stochastic Flows of Automorphisms of G-structures of Degree r*

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1. Introduction. Let M be a σ-compact connected C^∞ manifold of dimension n, and let $P^r(M)$ be the bundle of frames of r-th order contact over M with structure group $G^r(n)$ and natural projection π ([6], [7]). The purpose of this paper is to give a condition that a stochastic flow of diffeomorphisms generated by a stochastic differential equation on M be a stochastic flow of automorphisms of a G-structure of degree r (i.e. a G-subbundle of $P^r(M)$) for a closed subgroup G of $G^r(n)$ by using Itô's formula for fields of geometric objects ([1]). Our main result (Theorem 3.1) generalizes some results in [4], [9], [1] on stochastic flows of diffeomorphisms leaving tensor fields invariant.

We assume all non-probabilistic maps (and vector fields) in this paper are smooth. The tangent bundle over M is denoted by $T(M)$, and the tangent space at $x \in M$ by $T_x(M)$. Throughout the paper, indices take the following values: $\alpha, \beta = 1, 2, \cdots, k$; $\lambda = 0, 1, \cdots, k$.

2. Preliminaries. Let M and G be as above. The quotient space $P^r(M)/G$ is then a fiber bundle with standard fiber $G^r(n)/G$ associated with $P^r(M)$. There is a natural one-to-one correspondence between the sections $M \to P^r(M)/G$ and the G-structures of degree r on M ([8, pp. 57–58]). A transformation φ of M induces a transformation $\tilde{\varphi}$ of $P^r(M)$ by $\tilde{\varphi}(j_0^r(f)) = j_0^r(\varphi \circ f)$ for any $j_0^r(f) \in P^r(M)$, where $j_0^r(f)$ is the r-jet at the origin $0 \in \boldsymbol{R}^n$ given by a diffeomorphism f of an open neighborhood of $0 \in \boldsymbol{R}^n$ onto an open set of M with $\pi(j_0^r(f)) := f(0)$. Then $\tilde{\varphi}$ induces a transformation $\bar{\varphi}$ of $P^r(M)/G$ such that $\bar{\varphi} \circ \mu = \mu \circ \tilde{\varphi}$, where $\mu: P^r(M) \to P^r(M)/G$ is the projection. For a section $\sigma: M \to P^r(M)/G$, we define a section $\varphi^\# \sigma: M \to P^r(M)/G$ by $\varphi^\# \sigma = \bar{\varphi}^{-1} \circ \sigma \circ \varphi$.

Correspondingly, a vector field $X: x \mapsto X(x) \in T_x(M)$, $x \in M$, on M induces a vector field $\tilde{X}$ on $P^r(M)$ and a vector field $\bar{X}$ on $P^r(M)/G$ in a natural manner, since X generates a local one-parameter group of local transformations φ_t of M and φ_t induces naturally a local one-parameter group of local transformations $\tilde{\varphi}_t$ [resp. $\bar{\varphi}_t$] of $P^r(M)$ [resp. $P^r(M)/G$]. We set $\varphi_t^\# \sigma = (\bar{\varphi}_t)^{-1} \circ \sigma \circ \varphi_t$. The vector field $\tilde{X}$ [resp. $\bar{X}$] is called the *natural lift of X to $P^r(M)$* [resp. $P^r(M)/G$]. We denote by $\hat{L}_X \sigma: M \to T(P^r(M)/G)$ the *Lie derivative of σ with respect to X in the sense of Salvioli* ([10]); it is defined by