24. *Convergence of a Class of Discrete Cubic Interpolatory Splines*

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(Communicated by Shokichi IYANAGA, M. J. A., March 12, 1990)

1. Introduction. Discrete cubic splines which interpolate given functional values at one point lying in each mesh interval of a uniform mesh have been studied in [2]. The case in which these points of interpolation coincide with the mesh points of a nonuniform mesh was studied earlier by Lyche [4], [5]. For further results in this direction reference may be made to Dikshit and Rana [3]. In order to obtain the sharp convergence properties, we study in the present paper the problem of one point interpolation by discrete splines when the interpolatory points are not necessarily equispaced. The results, obtained in this paper include in particular some earlier results due to Lyche [5] for uniform mesh, Dikshit and Powar [2] and Chatterjee and Dikshit [1].

2. Existence and uniqueness. Let P: $a=x_0<x_1<\cdots<x_n=b$ denote a partition of $[a, b]$ with equidistant mesh points so that $p=x_i-x_{i-1}$ for all i. For a given $h>0$, suppose a real function $s(x, h)$ defined over $[a, b]$ and its restriction on $[x_{i-1}, x_i]$ is a polynomial s_i of degree 3 or less for $i=1, 2, \cdots, n$. Then $s(x, h)$ defines a discrete cubic spline if

(2.1) $$(s_{i+1}-s_i)(x_i+jh)=0,\ j=-1, 0, 1;\ i=1, 2, \cdots, n-1.$$

For an equivalent definition of a discrete cubic spline we introduce the difference operator

$$D_h^{\{0\}}f(x)=f(x);\ D_h^{\{1\}}f(x)=(f(x+h)-f(x-h))/2h;$$
$$D_h^{\{2\}}f(x)=(f(x+h)-2f(x)+f(x-h))/h^2.$$

We also use basic polynomials $x^{\{j\}}$ given by

$$x^{\{j\}}=x^j,\ j=0, 1, 2;\ x^{\{3\}}=x(x^2-h^2)$$

and observe that the condition (2.1) has the following equivalent form

(2.2) $$D_h^{\{j\}}s_i(x_i, h)=D_h^{\{j\}}s_{i+1}(x_i, h),\ j=0, 1, 2;\ i=1, 2, \cdots, n-1.$$

The class of all discrete cubic splines on P is denoted by $D(3, P, h)$ whereas $D_1(3, P, h)$ denotes the class of all $b-a$ periodic discrete cubic splines of $D(3, P, h)$.

We suppose that $\langle\theta_i\rangle_{i=1}^{\infty}$ is a real periodic sequence with period n so that $\theta_i=\theta_{i+n}$, $i=1, 2, \cdots$. Considering the points $y_i=x_{i-1}+\theta_i p$, $0\leq\theta_i\leq 1$, $i=1, 2, \cdots, n$, we propose the following.

Problem 1. Given $h>0$, for what restrictions on $\langle\theta_i\rangle$ does there exist a unique spline $s(x, h)\in D_1(3, P, h)$ satisfying the interpolatory condition

(2.3) $$s(y_i, h)=f(y_i),\ i=1, 2, \cdots, n,$$

where $\{f(y_i)\}$ is a given sequence of functional values?