73. A Uniqueness Set for Linear Partial Differential Operators with Real Coefficients

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1. Introduction. Let d be a positive integer and $d \geqslant 2$. $\mathcal{O}(C^d)$ denotes the space of holomorphic functions on C^d. Suppose P is an arbitrary irreducible homogeneous polynomial with real coefficients. For any complex number λ we put $\mathcal{O}_\lambda(C^d)=\{F \in \mathcal{O}(C^d)\,;\ (P(D)-\lambda)F=0\}$. Let $\mathcal{N}=\{z \in C^d\,;\ P(z)=0\}$. The space $\mathcal{O}(\mathcal{N})$ of holomorphic functions on the analytic set $\mathcal{N}$ is equal to $\mathcal{O}(C^d)|_{\mathcal{N}}$ by the Oka-Cartan theorem.

Consider the restriction mapping $\alpha_\lambda : F \to F|_{\mathcal{N}}$ of $\mathcal{O}_\lambda(C^d)$ to $\mathcal{O}(\mathcal{N})$. In our previous paper [5] we showed that α_λ is a linear isomorphism of $\mathcal{O}_\lambda(C^d)$ onto $\mathcal{O}(\mathcal{N})$ when $P(z)=z_1^2+\cdots+z_d^2$ $(d \geqslant 3)$. In this sense we called the cone $\{z \in C^d\,;\ z_1^2+\cdots+z_d^2=0\}$ a uniqueness set for the differential operator $\sum_{j=1}^d (\partial/\partial z_j)^2+\lambda^2$ (for the case $P(z)=z_1^2+\cdots+z_d^2$, see also [4] and see [3] for more general polynomials of degree 2).

In this paper we will show that α_λ is a linear isomorphism of $\mathcal{O}_\lambda(C^d)$ onto $\mathcal{O}(\mathcal{N})$ for any $\lambda \in C$ if P is an arbitrary irreducible homogeneous polynomial with real coefficients.

2. Statement of the result and its proof. Let P be an arbitrary homogeneous polynomial and we define the polynomial P^* by $P^*(z)=\overline{P(\bar{z})}$ $(z \in C)$. $P(C^d)$ denotes the space of polynomials on C^d and $H_k(C^d)$ denotes the space of homogeneous polynomials of degree k on C^d. We define the inner product $\langle\ ,\ \rangle$ on $P(C^d)$ by the following formula:

$$\langle z^\alpha, z^\beta \rangle = \begin{cases} 0 & (\alpha \neq \beta) \\ \alpha\,! & (\alpha=\beta). \end{cases}$$

We put $\mathcal{H}_k=\{F \in H_k(C^d)\,;\ P^*(D)F=0\}$ and $J_k=\{P\phi \in H_k(C^d)\,;\ \phi$ is some homogeneous polynomial on $C^d\}$. The following lemma is known.

Lemma 2.1 ([1] and [2] Theorem 3). (i) For any nonnegative integer k we have $H_k(C^d)=\mathcal{H}_k \oplus J_k$ and $\mathcal{H}_k \perp J_k$ with respect to the inner product $\langle\ ,\ \rangle$.

(ii) For any $\lambda \in C$ and any $F \in \mathcal{O}(C^d)$ there exist $H, G \in \mathcal{O}(C^d)$ uniquely such that

$$F=H+PG \tag{2.1}$$

and

$$(P^*(D)+\lambda)H=0. \tag{2.2}$$

Suppose $F \in \mathcal{O}(C^d)$. Let $F(z)=\sum_{k=0}^\infty F_k(z)$ be the development of F in a series of homogeneous polynomials F_k of degree k. Then $\sum_{k=0}^\infty F_k$ converges