76. *First Order Rational Differential Equations Depending Transcendentally on Arbitrary Constants*

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0. In [3] Painlevé gives an example of first order rational differential equation whose general solution depends transcendentally on arbitrary constants. This example, however, as will be seen in later, is defined essentially over the complex number field. The aim of this note is to get an example defined over the field containing nonconstant functions without separable variables. To this end it will be necessary to seize some of notions introduced by Painlevé from the viewpoint of differential algebra.

Let K be a differential field of characteristic 0 with a single differentiation $'$. In what follows every differential field extension of K will be regarded as differential subfields of a fixed universal differential field extension of K. Let R be a differential field extension of K and a finitely generated field extension of K. We say that R depends algebraically on arbitrary constants if there exists a differential field extension E of K such that E and R are free over K and $m(R:E)=[ER:EC_{ER}]$ is finite, where C_L for a differential field L denotes the field of constants of L. If this is case, by $m(R)$ we denote the minimum of such numbers $m(R;E)$. Then there exists an intermediate differential field S between R and K such that $m(R)=[R:S]$ and $m(S)=1$ provided K is algebraically closed (see [2]). We remark that if we consider a new differentiation $*$ in R by $u^*=au'$ for any u in R with a fixed nonzero a in K the property of algebraic dependence on arbitrary constants will be left unaltered, because in the above definition to be constant with respect to $'$ is the same as be so with respect to $*$. The number $m(R)$ corresponds to the number of branches of general solution around a movable singularity which was investigated by Painlevé.

1. **Lemma.** *Let R be a differential algebraic function field of one variable over K. If there exists a finite chain of differential field extensions of K: $K=F_0\subseteq F_1\subseteq\cdots\subseteq F_m$ such that $R\subseteq F_m$ and for each i F_i is algebraic extension of F_{i-1} or a differential algebraic function field over F_{i-1} depending algebraically on arbitrary constants then R depends algebraically on arbitrary constants.*

Proof. Let m be the minimum index for which R is contained is some finite algebraic extension F of F_m. Then F is a differential algebraic function field over F_{m-1} depending algebraically on arbitrary constants. Hence there exists a differential field extension E of K such that E and F are free over K and $m(F:E)$ is finite. Since C_{EF} and ER are linearly disjoint over