43. *Estimates for Degenerate Schrödinger Operators and an Application for Infinitely Degenerate Hypoelliptic Operators*

By Yoshinori MORIMOTO
School of Mathematics, Yoshida College, Kyoto University

(Communicated by Kôsaku YOSIDA, M. J. A., June 13, 1989)

1. Introduction and main theorems. In Chapter II of [1] Fefferman and Phong estimated the eigenvalues of Schrödinger operators $-\Delta+V(x)$ on R^n by using the uncertainty principle. Inspirated by their idea, in the present note we give two L^2-estimates for degenerate Schrödinger operators of higher order, which are a version and an extension of Theorem 4 in Chapter II of [1]. As an application, we consider the hypoellipticity for an example of infinitely degenerate elliptic operators.

Consider a symbol of the form

$$a(x, \xi)=\sum_{k=1}^{n} a_k(x)|\xi_k|^{2\mu_k}+V(x), \qquad x \in R^n, \tag{1}$$

where μ_k are positive rational numbers, $V(x)$ is a non-negative measurable function and

$$\begin{cases} a_1(x)=1, \\ a_k(x)=\prod_{j=1}^{k-1} |x_j|^{2\kappa(k,j)} \qquad \text{for } k\geq 2. \end{cases} \tag{2}$$

Here $\kappa(k, j)$ are non-negative rational numbers. If $(x_0, \xi_0) \in R^{2n}$ and if $\delta=(\delta_1, \cdots, \delta_n)$ for $\delta_j>0$, we denote by $B_\delta(x_0, \xi_0)$ a box

$$\{(x, \xi)\,;\, |x_j-x_{0j}|\leq \delta_j/2,\ |\xi_j-\xi_{0j}|\leq \delta_j^{-1}/2\}. \tag{3}$$

Clearly the volume of $B_\delta(x_0, \xi_0)$ is equal to 1. Let $\mathcal{C}$ denote a set of boxes $B_\delta(x_0, \xi_0)$ for all (x_0, ξ_0) and all δ. We denote by $\mathrm{m}_l(\cdot)$ the Lebesgue measure in R^l. We set $m_k=\mu_k-1$ if μ_k is integer and $m_k=[\mu_k]$ otherwise. Set $m_0=\sum_{k=1}^{n} m_k$.

Theorem 1. *Let $a(x, \xi)$ be the above symbol and let $W(x)$ be a continuous function in R^n. Assume that there exists a constant $1-2^{-m_0}<c\leq 1$ such that for any $B=B_\delta(x_0, \xi_0) \in \mathcal{C}$*

$$\mathrm{m}_{2n}(\{(x, \xi) \in B\,;\, a(x, \xi)\geq \max_{\pi(B^{**})} W(x)\})\geq c, \tag{4}$$

where π is a natural projection from $R^{2n}_{x,\xi}$ to R^n_x and B^{**} denotes a suitable dilation of B whose modulus depends only on μ_k and $\kappa(k, j)$. Then for any compact set K of R^n_x there exists a constant $c_K>0$ such that

$$(a(x, D)u, u)\geq c_K(W(x)u, u) \qquad \textit{for any } u \in C_0^\infty(K), \tag{5}$$

where $(\ ,\ )$ denotes the L^2 inner product (cf. Theorem B in [5]).

Remark 1. The lower bound of c in (4) is 0 when all $\mu_k\leq 1$. If all $a_k(x)\equiv 1$ then the constant c_K in (5) can be taken independent of K. The